Remember to *present* your solutions to the problem in words. Another student should be able to look at your homework page and be able to figure out what the question was asking without looking at this sheet. please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

1) PhET Simulation – Air Resistance

In your textbook (section 2.4), Taylor solves for the case of a baseball being dropped from a high tower using quadratic air resistance, $\vec{F}^{drag} = -cv^2\hat{v}$. Lets look at the case of a ball being shot *up* at an initial speed v_0.

(a) Draw a free body diagram for a ball moving vertically upwards, subject to quadratic air drag. Write down a differential equation for this situation, and solve this differential equation for $v_y(t)$. Make a rough sketch of $v_y(t)$ vs. t, and briefly discuss any key features.

(b) Using your result from the previous part, find an expression for the time it takes to reach the top of the trajectory. (It will look simpler if you write it in terms of terminal velocity, which satisfies $v_{ter}^2 = mg/c$.

(c) Now run the PhET simulation (https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html) and click on the "Lab" tab. On the upper right, switch the object to "baseball". This simulation uses quadratic drag: $\vec{F}^{drag} = -\frac{1}{2}c_D\rho_{air}Av^2\hat{v}$, where c_D is called the drag-coefficient, A is the cross-sectional area of the object being shot, and ρ_{air} is the density of air = 1.3 kg /m^3. (The simulation shows you the value of c_D and diameter it has picked for a baseball, on the right side of the simulation.) Use your formula in part (b) for "time to top" with these numbers to deduce what numerical initial velocity v_0 you need to get the ball to reach the top of its trajectory at precisely $t = 2.5$ sec. Now test it; you can input v_0 into the simulation and fire the cannon. Aim the cannon at 90° (or 89° if it is easier to see the trajectory) and switch on air resistance. The little + and – glasses let you zoom in or out. Does the ball reach the top at $t = 2.5$ sec? (It should!)

(d) When you fired the ball on the PhET simulation, did it take longer for the ball to go from the ground to the top of the trajectory or from the top of the trajectory to the ground? Explain why this is the case.

(e) Now let's look at another interesting feature of shooting an object up in the air. Playing with the simulation, write down the initial velocity that makes the ball reach the top of its trajectory at 4 sec, then 5 sec, then 6 sec, and so on. What do you notice happening? Make a plot of t_{top} vs. v_0 and explain in words how this relates to what you see on the simulation.

Extra credit

Explore with the PhET simulation a little more and study deeper anything you are interested in. Write down one question that you have about what you notice while exploring with the simulation.

2) John Taylor's range approximation

(a) In Taylor's book, he simplifies Eqn 2.42 for the range of a projectile by assuming that $R^2 \sim R_{vac}^2$. Show that if instead you use the quadratic equation to solve for R directly in Eqn 2.42, and make the appropriate Taylor series expansion, that you obtain the expression given in Equation 2.44.

(b) If you use your result from the previous part to compute R when $v_{y0} = v_{ter}$, what do you get? Does this make sense? (Briefly, discuss)

3) Romeo and Juliet

Romeo and Juliet are in love, but in our version of this story, the characters are a little different than tradition holds. Let $R(t)$ = Romeo's feelings for Juliet at time t. Large positive values means deep love, large negative values means strong hatred. Similarly, Juliet's feelings for Romeo will be characterized by $J(t)$. Now let's consider a "model for the lovers" governed by the general linear coupled differential equations,

$$\frac{dR(t)}{dt} = a\,R(t) + b\,J(t) \tag{1}$$

$$\frac{dJ(t)}{dt} = c\,R(t) + d\,J(t) \tag{2}$$

If $a > 0$ and $b = 0$, I might describe Romeo as "in love with his love". His emotional state has nothing to do with Juliet's feelings at all; he just falls more in love the more in love he is … a very narcissistic individual!

(a) Let's assume none of the parameters are zero. Consider *all four* possible permutations of signs of a and b, and for each, describe the emotional character of Romeo. (For instance, one of the combinations might be called a "emotionally distant jerk", which one? But maybe you have a better or totally different description/interpretation of this same combination!)

(b) Suppose that Juliet has the exact same *emotional character* as Romeo. (I'm not saying $J(t) = R(t)$, but they respond to each other in analogous ways.) What would this say about parameters c and d in terms of a and b? In this case are there values for the parameters a and b that would lead to equilibrium (*i.e.*, all time derivatives vanish)? Is there any equilibrium solution for $J(t)$ and $R(t)$ besides "complete mutual indifference", *i.e.*, $R = J = 0$?

You might be wondering why I am asking you to think about such a whimsical question like this in Phys 311. *This question is really just about developing intuitions about ODEs. Physics is full of ODEs! You will need such intuitions in almost every branch of physics. I got the question from Strogatz' "Nonlinear Dynamics and Chaos" text.*

4) Determining the equation of motion — Gravitational force and air drag

Consider a ball that moves vertically under the influences of both the gravitational force and air resistance. For the purposes of this problem, take vertically upward as the positive direction. For each equation of motion below, determine whether that equation applies to (i) a situation in which the ball moves upward, (ii) a situation in which the ball moves downward, (iii) either of these, or (iv) neither of these. Explain your reasoning for each case.

(a) $m\frac{dv_y}{dt} = -mg + bv_y$

(b) $m\frac{dv_y}{dt} = -mg - bv_y$

(c) $m\frac{dv_y}{dt} = -mg + cv_y^2$

(d) $m\frac{dv_y}{dt} = -mg - cv_y^2$

5) Time of flight with linear drag

You're hanging out by a campfire after acing the first Phys 311 exam. You notice a tiny piece of fly ash that is ejected from the fire vertically upward with initial speed v_0. Since physicists tend to love to think about casual observations like this more formally, you decide to try to predict if the ash will reach tree-level. Let's measure the position of the fly ash, y from the point of release, once again taking vertically upward as the positive direction.

(a) Clearly the physics of "flying ash" could be very complicated! But, let's start with the *simplest possible* model and go from there. If you consider ash to be a point particle of mass m acted on by the gravitational force only, what is the equation of motion? What is the maximum height (in terms of v_0 and g)?

(b) The next simplest model might be to also include simple air drag. Taylor discusses the choice between "linear" and "quadratic" drag. Knowing that ash particles are very small, and not particularly fast moving to start with, if you had to choose just ONE of these, which should it be? Briefly (qualitatively is all I want) justify your answer.

(c) Now, using this model, find the time for the fly ash to reach its highest point and its position y_{max} at that time. (Note that velocity $v_y(t)$ and position $y(t)$ are derived in the text. Of course, it's always a good idea to rederive them yourself, but not required for credit. However, you should be careful, because in the text, Taylor defines $+y$ to be down, so watch your minus signs! I suggest using notation from question 1 to define symbols. No numbers here, do it all symbolically.)

(d) Show that as the drag coefficient approaches zero your answer in part (c) reduces to the well known freshman physics result you got in part (a). Hint: If the drag coefficient is small the terminal velocity is big, so v_0/v_{ter} is very small. Use the Taylor expansion $\ln(1+\varepsilon) \approx \varepsilon - \varepsilon^2/2 + \cdots$ for ε close to zero.

6) Particle sliding down a plane with air resistance

A grain of pollen (mass m) slides down a smooth, flat, tilted solar panel (tilted at θ from horizontal) under the influence of the gravitational force.

(a) If the motion is resisted by a linear drag force $F^{drag} = kmv$ find an expression for how far down the plane the grain slides in time t, assuming it is released from rest. ***Hint***: Draw a free body diagram to start!

(b) Show that as the drag coefficient becomes very small, your answer in part (a) reduces to a simple freshman physics result (which is what?). Comment briefly on the similarities and differences between this problem and question 2 on Homework 2. ***Hint***: Here, I claim the useful Taylor expansion you'll want will be $e^x \approx 1 + x + x^2/2 + \cdots$, *for x* close to 0. And yes, you'll really need to go all the way out to that $x^2/2$ term!

For you to think about. You do not have to write this up for credit, but we'll revisit this idea again and again; it's very important! The power x in e^x must be unitless. The "Drag coefficient" has units. So it can not be exactly correct to say the "drag coefficient becomes small", without answering "small compared to what"? What exactly is it that is really the "small thing, x" in this case?

7) Finding the range numerically (Air Resistance)

Consider a ball thrown at an angle θ above the horizontal ground with an initial speed v_0 in a medium with linear drag. For numerical solutions, computational tools don't deal well with unknown symbols, so let's consider a particular case where $v_0 = v_{ter}$ and $v_{ter}^2/g = 1\,\text{m}$. We know that in vacuum the maximum range is at $\theta = \pi/4$. Let's try to estimate the maximum range (and angle) when we include air resistance:

(a) Plot Eqn. (2.37) in Taylor for different values of θ and try by inspection to figure out the angle at which the range is maximum. (With the assumptions above, most unit-full quantities become "one", but do be careful of the fact that. *e.g.*, v_{0x} is $v_0 \cos(\theta)$, not v_0), etc. Briefly comment on real-world implications!

(b) Use a root finder to find the range when $\theta = \pi/4$. Any root finder will require you to make a guess for the root. This is what is called the "neighborhood of the root" or "bracketing the root". A screencast showing how to use *Mathematica* to find roots is available here: https://www.youtube.com/user/compphysatcu/videos

(c) Repeat for different values of θ (homing in on a small range near the angle you estimated in part (a)). Continue until you know the maximum range and the corresponding angle to two significant figures. Connect to point a) and compare with the ideal vacuum values. Briefly, discuss.
To consider: Check what happens in the limit $v_0 \gg v_{ter}$ (say $v_0 = 100 v_{ter}$) and $v_0 \ll v_{ter}$ (say $v_0 = 0.01 v_{ter}$). Discuss if your results make sense.

8) Golfing on the Moon

Find the regulation size (and mass, while you're at it) for a golf ball.

(a) The drag coefficient c_D for a golf ball traveling in air is (very roughly) 0.25. (Note: the c in cv^2 is given by $c = \frac{1}{2} c_D \rho_{air} A$.) Calculate the numerical value of the quadratic drag constant "c" for a golf ball traveling in air, and write down the equations of motion required to solve for $x(t)$ and $y(t)$ of a golf ball with (just) quadratic drag.

(b) In class we did a *Mathematica* Tutorial using NDSolve. Modify your code so that you have *quadratic* drag. For simplicity and concreteness, set the angle to 45 degrees, and pick an initial speed of 80 mi/hr (converted to SI metric, of course). Plot the trajectory for us. What is the ideal (drag-free) range in this case, and what does your code tell you the range is when including quadratic drag? (Don't hunt for the optimum range, just stick with a 45 degree angle) Also, calculate what fraction of the initial *speed* is lost from when you first hit the ball (at 45 degrees) to when it lands. (Where does the "lost" energy go?)

(c) When I first started thinking about this problem, I wondered whether the fractional reduction in speed was a constant, or whether it depended on the initial velocity. What do you think? (No formal calculation required, just make a qualitative physics argument.) *Check* yourself with your code, and comment.

(d) Astronaut Alan Shepard on Apollo 14 brought a golf club with him to the Moon (!) Assuming he hit it at 45 degrees (and assuming he hit it at that same 80 mi/hr, a modest swing on Earth, maybe optimistic on the Moon, given his clumsy space suit), calculate how far the ball landed from him, and compare to what it would have been on Earth. (Which effect is more significant in the end, the loss of air drag on the moon, or the difference in the local gravitational constant?)