Remember to *present* your solutions to the problem in words. Another student should be able to look at your homework page and be able to figure out what the question was asking without looking at this sheet. please show your work and explain your reasoning. I will grade for clarity of explanation as much as I do for mere "correctness of final answer"!

Choose 1 problem from problems 1–2 to complete.
Choose 1 problem from problems 3–4 to complete.
Choose 1 problem from problems 5–6 to complete.

1) Center of Mass — Triangular plate

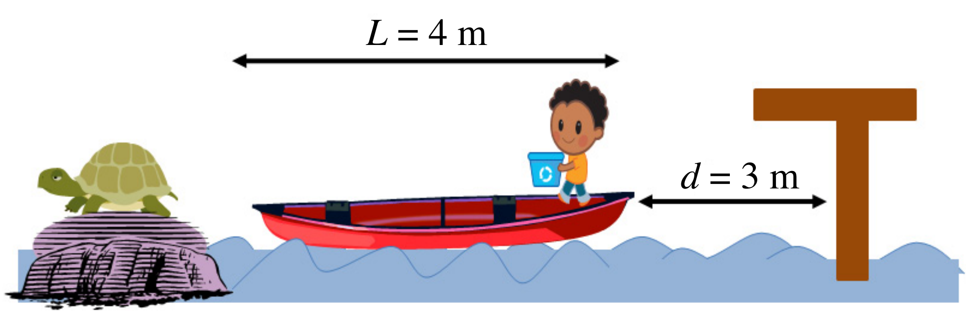
You have a flat plate (of uniform mass density, which is mass per unit *area* in this case) of total mass M. It is in the shape of a right triangle, with legs a and b (c is the hypotenuse).

(a) Draw your triangle, choosing a coordinate system origin and orientation to make it as easy as possible for you to find the center of mass. Based on your physical and mathematical intuitions, without calculating, where do you predict the center of mass is, and why?

(b) Now mathematically determine its center of mass coordinates. Does your answer match your intuitions from part (a)? Briefly, explain (or reconcile!)

2) Center of Mass — "Walking" on water

Young Peter (mass m_P) is standing at one edge of a 4 m long canoe (mass M_c). He observes a turtle on top of a rock that is close to the other edge of the canoe. Peter wants to catch the turtle and starts walking towards it. You may ignore any water friction.



(a) Qualitatively describe the motion of the system (canoe+ Peter) as Peter walks forward. If initially the canoe is 3 m away from the dock, where is Peter with respect to the dock when he reaches the other end of the canoe? (I would like an expression in terms of m_P, M_c, L, and d.)

(b) If $m_P = 40$ kg and $M_c = 30$ kg and Peter can stretch his arm 1 m away from the canoe edge, can he catch the turtle? What happens in the limit of a very light canoe? A very massive canoe? (Light or massive compared to what? It might help to write your result from (a) in terms of the ratio m_P/M_c or M_c/m_P)

3) Aircraft crash

Two FBI agents (let's call them Mulder and Scully) are investigating the wreckage of a drone aircraft, which is in three large pieces around a northern Colorado town. One piece (mass = 300 kg) of the aircraft landed 6.0 km due north of the center of town. Another piece (mass = 1000 kg) landed 1.6 km to the southeast (36 degrees south of east) of the center of town. The last piece (mass = 400 kg) landed 4.0 km to the southwest (65 degrees south of west) of the center of town. There are no more pieces of the drone aircraft. The Air Force, which was watching the drone aircraft on its radar, claims it was moving with a constant speed of 5 m/s to the east at a height of 1.96 km. The aircraft was 100 m west of the center of town when the explosion happened and the pieces fell to the ground. There is also evidence that none of the pieces acquired appreciable vertical velocities immediately after the explosion. Agents Mulder and Scully think a missile hit the aircraft. Are the fragments consistent with the drone aircraft exploding spontaneously? If not, can you tell what direction the missile came from?

4) Multi-stage rockets

A rocket having two or more engines, stacked one on top of another and firing in succession is called a multi-stage rocket. Normally each stage is jettisoned after completing its firing. The reason rocketeers stage models is to increase the final speed (and thus, altitude) of the uppermost stage. This is accomplished by dropping unneeded mass throughout the burn so the top stage can be very light and coast a long way upward. Let us understand better the advantages of a multi-stage rocket. Imagine that the rocket carries 80% of its initial mass as fuel (*i.e.*, the mass of all the fuel is $0.8m_0$).

(a) What is the rocket final speed accelerating from rest in free space, if it burns its fuel in a single stage? Express your answer in terms of v_{ex}

(b) Now suppose instead that it burns the fuel in two stages like this: In the first stage it burns a mass $0.4m_0$ of fuel. It then jettisons the (empty) first stage fuel tank. Let's assume this empty tank has a mass of $0.1m_0$. It then burns the remaining $0.4m_0$ of fuel. (So, we've burned the same total amount of fuel as part (a), right? We simply jettisoned an empty fuel-stage in the middle) Find the final speed in this case, assuming the same value of v_{ex} as in part (a). Compare and discuss briefly.

5) Hovering rockets

Taylor worked out the rocket equation in deep space. But at launch, obviously you cannot neglect the gravitational force – the net external force, and hence $d\vec{p}/dt$, is no longer zero.

(a) Follow Taylor's derivation on page 86, and fix it up, getting to Equation 3.6, and find the "correction" term you need to add to include the gravitational force. Assuming that v_{ex} is a fixed (constant) number, and assuming that you want the rocket to simply "hover" above the ground (rather than really launching), analytically solve the ODE you get to find rocket mass as a function of time. (Does dm/dt turn out to be a constant? Explain physically why your answer to that question makes sense.)

(b) If your payload (the mass that is left over after all the fuel is gone) is roughly $e^{-2} = 0.135$ of the initial mass, how long can you hover? Given the (very optimistic!) value of $v_{ex} = 2000$ m/s, comment on why we don't all commute around with jetpacks.

6) Rocket with a linear drag force

So far we have considered the ideal case of a rocket without drag. In real life, however, drag can be an important limitation and must be considered. Imagine the situation of a linear drag force, $\vec{F}^{drag} = -b\vec{v}$, acting on the rocket body only (with no other external forces, so we're back to the "gravity free" case of deep space)

(a) Once again, the net external force, and hence $d\vec{p}/dt$, is no longer zero. Follow Taylor's derivation on page 86, and fix it up, getting to Equation 3.6, and find the "correction" term you need to add to include the drag force.

(b) Now solve your ODE, to show that if the rocket starts from rest and ejects a mass at constant rate $\dot{m} = -k$ (with k a given positive constant), then its speed is given by $v = \frac{k}{b} v_{ex} \left[1 - \left(\frac{m}{m_0} \right)^{\lambda} \right]$. What is λ in terms of k, b, v_{ex}, and m_0?

As a check, you could put your ODE into Mathematica, pick some reasonable numbers, and simply plot v as a function of m.
Hint: since $dm/dt = -k$, you can eliminate any stray "dt" terms that appear in your ODE.

(c) What is the corresponding speed if we ignore drag? Show that the equation above reproduces the speed for the drag free case if $b \to 0$ (see hint below). Calculate the first non-vanishing correction introduced by a finite drag force to the speed. Does the sign of your correction make physical sense? Briefly, discuss.

Hint: A toolbox math technique that may be helpful here: you can always rewrite the function $f(x) = c^x$ as $f(x) = e^{\ln(c^x)} = e^{x\ln(c)}$.