Vector Methods

Why be familiar with vector methods?

– Coordinate transformations

– Matrices and their use in transformations

– Vector Addition & Subtraction $\vec{A}+\vec{B}$, $\vec{A}-\vec{B}$

– Vector Multiplication

1) w/ scalar $s\vec{A}$

2) scalar product (inner product) $\vec{A}\cdot\vec{B}=A_i B_i$

3) vector product $\vec{A}\times\vec{B}=\varepsilon_{ijk}A_j B_k$

4) outer product $A_i B_j$

– <u>Vector Calculus</u>

1) Scalar derivative $\frac{d\vec{A}}{ds}$

2) Gradient $\vec{\nabla}\phi$

3) Divergence $\vec{\nabla}\cdot\vec{A}$

4) Curl $\vec{\nabla}\times\vec{A}$

5) Laplacian $\nabla^2\phi$

6) Scalar Integral $\int \vec{A}dv$

7) Line Integral $\int \vec{A}\cdot d\vec{s}$

Surface Integral $\int \vec{A}\cdot\hat{n}da$

8) Gauss' theorem $\int \vec{A}\cdot\hat{n}da = \int \left(\vec{\nabla}\cdot\vec{A}\right)dv$

Stokes' theorem $\int \vec{A}\cdot d\vec{s} = \int \left(\vec{\nabla}\times\vec{A}\right)\cdot\hat{n}da$

Use of Vectors (Tensors) and Vector methods allows:

- describing the problem without explicit reference to a particular coordinate system.
- "Ease" of transforming explicit results from one coordinate system to another.
- Compact, concise way of expressing complicated results.

My Notation

Arbitrary scalar quantity:	s
Arbitrary vector quantity:	$\vec{A}$, $\vec{B}$
A vector component:	A_i, A_j (arbitrary) A_1, A_3 (more explicit) A_x, A_θ (very explicit)
A unit vector:	$\hat{e}$
A unit vector component:	$\hat{e}_i$, $\hat{e}_k$ (arbitrary) $\hat{e}_1$, $\hat{e}_2$ (more explicit) $\hat{e}_y$, $\hat{e}_\theta$ (very explicit)
Matrix:	$\overline{A}$
Matrix components:	A_{ij}, A_{ik} (arbitrary) A_{13}, A_{32} (more explicit) A_{xx}, $A_{r\theta}$ (very explicit)

Summations

$$\sum_{j=1}^{3} \lambda_{ij} A_j = \lambda_{i1} A_1 + \lambda_{i2} A_2 + \lambda_{i3} A_3$$

We will write this as: $\lambda_{ij} A_j$ (implicit sum for doubled indices)

Exception: Matrix elements, λ_{ij}

$$\lambda_{ii} = \lambda_{11} \text{ or } \lambda_{22} \text{ or } \lambda_{33}$$

$$\neq \sum_{i=1}^{3} \lambda_{ii} = \lambda_{11} + \lambda_{22} + \lambda_{33}$$