Linear, 2nd order, non-homogeneous differential equation with constant coefficients.

$$a\,\frac{d^2y(x)}{dx^2}+b\,\frac{dy(x)}{dx}+c\,y(x)=f(x)$$

The solutions are given by:

$$y(x)=y_p(x)+y_c(x)$$

where $y_c(x)\equiv$ complementary solution to the homogeneous equation.

To find $y_p(x)$, we guess at a solution. The guess we make will depend on $f(x)$.

$\boldsymbol{f(x)}$	$\boldsymbol{y_p(x)}$
$P_n(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$	$x^s\left(A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0\right)$
$P_n(x)e^{\alpha x}$ (α = real or complex)	$x^s\left(A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0\right)e^{\alpha x}$
$P_n(x)e^{\alpha x}\cos(kx)$ or $P_n(x)e^{\alpha x}\sin(kx)$	$x^s\left[\left(A_nx^n+\cdots+A_0\right)e^{\alpha x}\cos(kx)+\left(B_nx^n+\cdots+B_0\right)e^{\alpha x}\sin(kx)\right]$

Here s is the smallest nonnegative integer ($s = 0, 1, 2$) which will insure that no term in $y_p(x)$ is a solution of the corresponding homogeneous equation.