The first three problems ask you to draw new conclusions about allowed wavefunctions and energy eigenvalues. None of these four problems require that much "ink on the page." They are thinkers, but you do not need to slog through as much mathematics as the last homework.

1) Minimum Energy

When considering solutions to the Time-Independent Schrodinger Equation, we noted that we have different forms of solutions for $\psi(x)$ depending upon if $E > V(x)$ or $E < V(x)$ in that region. However, there must be some *global minimum* value for $V(x)$, $V_{\min}$. Show that E must exceed $V_{\min}$ if the wavefunction is to be normalizable. *Hint:* Rewrite the time-independent Schrodinger equation in the form $\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}[V(x) - E]\psi$, and think about the shape of $\psi(x)$ under these conditions. Argue that such a function cannot be normalized.

Make sure your argument is specific that E cannot be less than the *minimum* $V(x)$, not just any $V(x)$. E *can* be less than $V(x)$ for *some* values of x, just not for *all* values of x.

2) Superposition

Show that if $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to the Schrodinger equation, then the superposition $\Psi(x,t) = c_1\Psi_1(x,t) + c_2\Psi_2(x,t)$ is a solution, too.

3) Real or Unreal Energy

Does the separation constant introduced on Page 26 of Griffiths (3rd edition, or Page 25 of 2nd edition), E, have to be a real number? Write E of Equation 2.7 as $E_r + iE_i$(as a sum of real and imaginary parts, where both E_r and E_i themselves are real). Show that if Equation 1.20 is to be true for all time, then E_i must be zero.

4) Infinite Square Well

Our goal here is to practice understanding the meaning of superposition and apply the formal mathematics of Fourier's trick in a situation where we can compare our answer to finding the superposition an "easier" way.

Consider a particle in a quantum mechanical infinite square well of width a. The wave function for this particle is given by $\psi_A(x) = \frac{4}{\sqrt{5a}}\sin^3(\frac{\pi x}{a})$.

(a) Does this wave function satisfy the boundary conditions for this potential? Explain.

(b) Write this wave function ψ_A as a superposition of the energy eigenfunctions ($\psi_n(x)$'s) of the infinite square well **by using trigonometric identities**.
Answer: $\psi_A = \frac{3}{\sqrt{10}}\psi_1 - \frac{1}{\sqrt{10}}\psi_3$

(c) Confirm that your answer to the previous part satisfies the condition $\sum_{n=1}^{\infty}|c_n|^2 = 1$.

(d) Write this wave function ψ_A as a superposition of the energy eigenfunctions ($\psi_n(x)$'s) of the infinite square well **using Fourier's trick**. Explain. You may (please do) use a computer to help with the integrals, but show how you set them up. (In other words, you are explicitly using Equation 2.37 to accomplish the same task as part (b) and demonstrate that you get the same answer.)

(e) Does the probability density of this particle in state ψ_A depend on time?

(f) If you measured the energy of this particle, what possible values could you get? How probable are they, or can you tell?