1) We Love Squares

For this problem, consider the **infinite square well potential** (from the previous section, Section 2.2). A particle of mass m in the infinite square well (of width a) starts out at time $t = 0$ in the left half of the well, and is equally likely to be found at any point in that region. (There is zero chance of finding it anywhere else.)

(a) Sketch $\Psi(x, 0)$ and $|\Psi(x, 0)|^2$. Label any values you can on the vertical and horizontal axes.

(b) Find $\Psi(x, t)$. *Hint: Use Fourier's Trick to find* c_n. *Justify why* $\Psi(x, t)$ *will require an* **infinite** *sum of stationary states.*

(c) What is the probability that a measurement of the energy would yield the value $\frac{\pi^2\hbar^2}{2ma^2}$?

We are practicing the fundamental task of Chapter 2: given a potential and an initial state, how do you find $\Psi(x, t)$? (*And how do you interpret the physical meaning of your result*?)

2) Stationary States of Harmonic Oscillator

Griffiths gives you the first two stationary states of the harmonic oscillator potential:

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \text{ (Equation 2.60)} \tag{1}$$

$$\psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{2m\omega}{\hbar}}\, x e^{-\frac{m\omega}{2\hbar}x^2} \text{ (Example 2.4)} \tag{2}$$

(a) Use a ladder operator to find $\psi_2(x)$. Don't forget your normalization factor; see Equation 2.68. Can you use the result from Example 2.4 to save time?

(b) Sketch $\psi_0(x)$, $\psi_1(x)$, and $\psi_2(x)$.

(c) Confirm that these three stationary states are all orthogonal to one another by explicit integration. *Hint:* Pay close attention to the even-ness or odd-ness of your functions from the sketches in part (b), and therefore also the even-ness or odd-ness of their products. In the end, there is only *one* integral left for you to do.

Here we are practicing using our newly defined operator, the ladder operator. We are also practicing our skills identifying useful symmetry and performing some of our favorite integrations.

3) Superposition

A particle in the harmonic oscillator potential starts out in the state:

$$\Psi(x,0) = A[3\psi_0(x) + 4\psi_1(x)] \tag{1}$$

(a) Find A.

(b) Construct $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Do either depend on time? Why? *Hint: Simplify using the identity* $e^{i\omega t} + e^{-i\omega t} = 2\cos(\omega t)$.

(c) Find $\langle x \rangle$. Does it depend on time? *Hint: Always be on the lookout for symmetry that lets you avoid actually doing integrals...*

(d) Find $\langle p \rangle$ *Hint: Don't define an integral at all. Find the easy way.*

(e) If you measured the energy of this particle, what values might you get, and with what probabilities?

(f) What is the expectation value of the energy? If I had a ensemble of 100 particles in this state, how many would yield measurements of energy equal to the expectation value?

This is our first time finding $\langle x \rangle$ *and* $\langle p \rangle$ *for a superposition state. One student will present parts a-c, the other will present parts d-f.*