1) Exam Questions

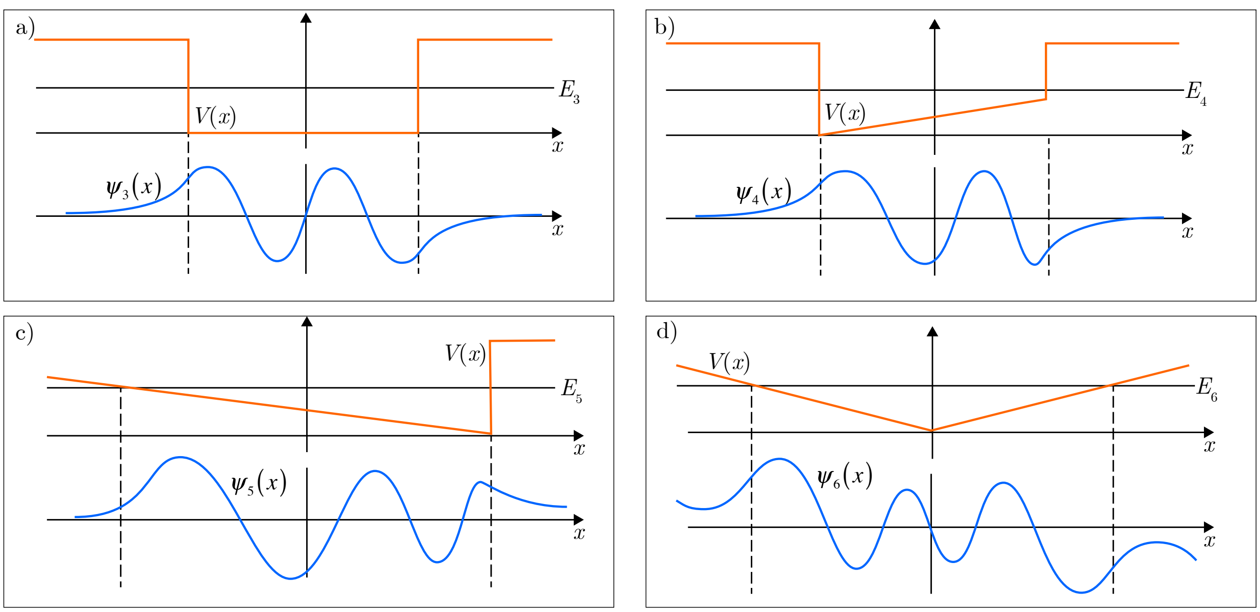
Create a list of the types of questions you expect to see on the first exam, which covers Chapters 1 and 2. Think about *verbs* and the *tasks* that you might be asked to do, *e.g.*, “Normalize a probability density". No one person will present this problem, but I expect everyone to come prepared to share a few from their list so that we build a full study guide.

We are focusing on qualitative properties of the solutions to the finite square well and similar potentials. You'll use the sketching rules given in our handout; though you aren't explicitly asked to do so, be sure you can explain why those rules are true. The last problem is quantitative, to allow you to practice finding separable solutions given a potential and using those solutions to discuss reflection and transmission probabilities.

2) Find the Errors

For parts a) through d) below, consider a particle whose potential energy is shown in each respective top graph. The wavefunction shown in the lower graph is supposed to be the energy eigenfunction corresponding to the third-, fourth-, fifth-, and sixth-lowest possible energy for the particle for parts a), b), c), and d), respectively.

What, if anything, is wrong with the eigenfunction as drawn? (Remember, an “energy eigenfunction” is another term for a “stationary state” or “separable solution” to the Schrodinger equation). Multiple things may be wrong, so be sure you list them all. Be sure to consider the curvature of the wavefunction (towards or away from the axis), the shape of the wave-like part (number of bumps, amplitude, wavelength) and the length of exponential tails.



3) Sketching I

The potential energy $V(x)$ for a particle is given by:

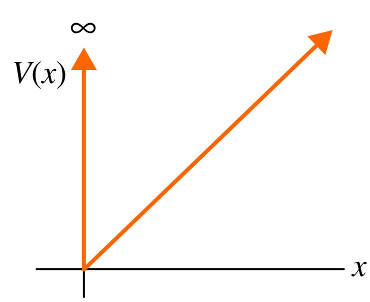
$$V(x) = \begin{cases} V_0 & \text{for } x < 0 \\ 0 & \text{for } 0 < x < a \\ V_0/2 & \text{for } a < x < 2a \\ V_0 & \text{for } x > 2a \end{cases}$$

(a) Sketch this potential.

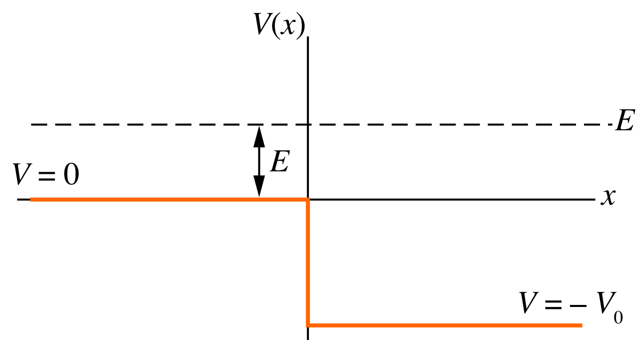
(b) Assume V_0 and a have been chosen so that $0 < E_1 < V_0/2 < E_2 < V_0$ for the energies E_1, E_2 of the first two stationary states. Without solving the TISE, draw two separate sketches, one for the ground state $\psi_1(x)$ with energy E_1, and one for the first excited state $\psi_2(x)$ with energy E_2. Comment on the features of the curves everywhere, including things like wavelength, curvature, and amplitude, and describe what happens at $x = 0, a, 2a$, using your qualitative knowledge of wave functions. (A sketch with no explanation will receive little or no credit!)

4) Sketching II

For the asymmetric well pictured below, sketch the states $\psi_1(x)$ (ground state), $\psi_2(x)$ (first excited state), and some larger $\psi_n(x)$ (like $n = 10$ or so). Explain in words the relevant important features of the wave functions, including things like wavelength, curvature, and amplitude. Also, describe a physical example of a potential energy function that looks like this.



5) Reflection off a down step



Consider a "down step" potential, which drops at $x = 0$ as one goes from left to right.

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ -V_0 & \text{for } x > 0 \end{cases}$$

where $V_0 > 0$.

(a) The first step for solving the TISE for fixed energy $E > 0$ in both regions is to write the general solution. Explain why this is the general solution:

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & \text{for } x < 0, \quad \text{where } k \equiv \sqrt{\dfrac{2mE}{\hbar^2}} \\ Fe^{ilx} + Ge^{-ilx} & \text{for } x > 0, \quad \text{where } l \equiv \sqrt{\dfrac{2m(E+V_0)}{\hbar^2}} \end{cases}$$

(b) Impose boundary conditions at $x = 0$; what does this tell you about the relationships between A, B, F; G, k, and/or l? Don't assume anything about where particles are coming from yet.

(c) Now, assume the particle is coming from the left. Calculate the reflection coefficient R and transmission coefficient T in terms of k and l. Check: Does this satisfy $R + T = 1$?

Answers: $R = \frac{k^2+l^2-2kl}{(k+l)^2}$ and $T = \frac{4kl}{(k+l)^2}$
Important Note: The transmission coefficient is a bit trickier here, because the incoming wave and the transmitted wave don't have the same speeds, so we can't just use the squares of two magnitudes to tell us the ratios of probabilities. Instead, we must use a definition using probability:

$$T = \frac{P_t}{P_i} = \frac{|\psi_t|^2 dx_t}{|\psi_i|^2 dx_i} = \frac{|\psi_t|^2 v_2 dt}{|\psi_i|^2 v_1 dt}$$

where the subscripts t and i stand for "transmitted" and "incident", respectively. Use what you know about the velocity of a traveling wave to express the ratio of the two velocities in terms of k and l.

(d) Write R and T in terms of the unitless V_0/E.

(e) Plot (or sketch) R vs. V_0 for a fixed E. Is the result surprising; is there any "quantum weirdness" here?