The first three problems focus on solutions to the Schrodinger equation in 3D Cartesian coordinates. The first gives you practice in using Separation of Variables and applying boundary conditions. Referencing Section 2.1 *may help remind you how we solve this problem in one spatial dimension. Though you are given the final answer, you of course must justify every step you take along the way to get there. The next two problems use this result to examine energy degeneracies. The final two problems are related to spherical coordinates.*

1) Infinite Cubical Well, Energy Eigenfunctions

Use separation of variables in Cartesian coordinates to find the stationary states of the infinite cubical well potential:

$$V(x,y,z) = \begin{cases} 0 & \text{if } x, y, \text{and } z \text{ are all between } 0 \text{ and } a \\ \infty & \text{otherwise} \end{cases}$$

Answer: $\psi(x,y,z) = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{n_x\pi}{a}x\right) \sin\left(\frac{n_y\pi}{a}y\right) \sin\left(\frac{n_z\pi}{a}z\right)$

2) Infinite Cubical Well, Energy Eigenvalues

(a) Show, using your results from the previous question, that for $n = 1,2,3...$

$$E = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2 + k_z^2\right) = \frac{\pi^2\hbar^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right) \tag{1}$$

(b) Call the distinct total energies $E_1,\ E_2,\ E_3$ ... in order of increasing energy. Find E_1 through E_6 (in terms of aforementioned constants). Is there a simple pattern, or is it more complicated than we saw for a particle in a 1D box? (Section 2.1)

(c) Determine the degeneracies of the aforementioned energy levels (that is, the number of different states that share the same energy). A “state” is one given combination of n_x, n_y, and n_z. In one dimension, degenerate bound states do not occur, but in three dimension they are very common. *Answer for* E_2*: degeneracy is* 3.

(d) What are the energies associated with the states with (n_x, n_y, n_z) = (3, 3, 3) and (5, 1, 1)? What is the total degeneracy of the (3, 3, 3) energy? (Note the question is not asking how many (3, 3, 3) states there are, it is asking how degenerate is the energy that goes with that state.)

3) Infinite Rectangular Cuboid Well

What if the particle is confined to a rectangular 3D box with edge lengths a, b, and c, which don’t have to be the same? In other words, the potential is:

$$V(x,y,z) = \begin{cases} 0 & \text{if } 0 < x < a \text{ AND } 0 < y < b \text{ AND } 0 < z < c \\ \infty & \text{otherwise} \end{cases}$$

(a) What is an expression for the energy of the (n_x, n_y, n_z) state? *This does rely on your answers to the previous questions.*

(b) Suppose the rectangular box has edges of length a, $b = 2a$, and $c = 3a$. What are the energies of the 5 lowest-energy states? Are any of these levels degenerate? In general, breaking symmetry tends to remove degeneracies in quantum mechanics; comment on how this statement applies to this problem.

4) Spherical Harmonics

(a) Use Equation 4.32 to construct Y_0^0 and Y_2^1.

(b) Check that both are normalized.

(c) Check that they are orthogonal to one another.

Here we are again practicing how to construct and work with the spherical harmonics. You can check answers to a) using Table 4.3. For b) and c), feel free to use WolframAlpha/Mathematica or make good use of tables of trigonometric identities and integrals.

5) Bad Solution

Griffiths Equation 4.25 is a second-order differential equation; it should have *two* linearly independent solutions, but you were only given one.

(a) Show that $\Theta(\theta) = A\ \ln[\tan(\theta/2)]$ also satisfies Equation 4.25 for $l = m = 0$.

(b) Why is this solution physically unacceptable?