Our problems here focus on using (and making physical sense of) the results of the derivation of ψ_{nlm} *for the hydrogen atom presented in Section* 4.2*. Use tables of integrals, especially one from the back cover of the book, or WolframAlpha/Mathematica; don't spend your time on the nitty-gritty of integration.*

1) Ground State of Hydrogen I

(a) Use Griffith's illustrations or our Mathematica visualization notebook to describe the probability density of the ground state of hydrogen, in words.

(b) What is the difference between "the expectation value of r" and "the most probable value of r," in general?

(c) What is the *most probable value* of r in the ground state of hydrogen? The answer is not zero! *Hints:* Recall that the probability for finding a particle in a certain volume is $|\psi|^2 d^3\vec{r}$, where $d^3\vec{r} = r^2 \sin\theta \, dr d\theta d\phi$. Find the probability that the particle will be found between r and $r + dr$. Find the location r where this function is maximized. Express your answer in terms of the Bohr radius, $a \equiv \frac{4\pi\epsilon_0\hbar^2}{me^2}$.

2) Ground State of Hydrogen II

(a) Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of hydrogen. Express your answer in terms of the Bohr radius, $a \equiv \frac{4\pi\epsilon_0\hbar^2}{me^2}$.

(b) Find $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of hydrogen. *Hint:* This requires no new integration. Use the fact that $r^2 = x^2 + y + z^2$, and exploit the symmetry of the ground state. Explain.

3) Excited State of Hydrogen

(a) Normalize R_{21} from Equation 4.83. Of course, you can check your answer by comparison to Table 4.7.

(b) Construct the wavefunction for the stationary state $n = 2,\ l = 1,\ m = 1$.

(c) Use Griffith's illustrations or our Mathematica visualization notebook to describe the probability density of this excited state, in words.

(d) Find $\langle x^2 \rangle$ for this state. Recall: $x = r \sin\theta \cos\phi$.

4) Superposition

A hydrogen atom starts out in the following linear combination of the stationary states $n = 2,\ l = 1,$ $m = 1$ and $n = 2,\ l = 1,\ m = -1$:

$$\Psi(\vec{r}, 0) = \frac{1}{\sqrt{2}} (\psi_{211} + \psi_{21-1}) \tag{1}$$

(a) Construct $\Psi(\vec{r}, t)$. Note you already constructed ψ_{211} for the problem above. Simplify as much as you can.

(b) Find the expectation value of the potential energy, $\langle V \rangle$. Does it depend on time? Give both the formula and the actual number, in electron volts.