

The Energy Eigenfunctions of the Simple Harmonic Oscillator $\left(\alpha^2 = \frac{m\omega}{\hbar}\right)$

n	E_n	$\psi_n(x)$
0	$\frac{1}{2}\hbar\omega$	$\left(\frac{\alpha^2}{\pi}\right)^{1/4} (1)e^{-\frac{\alpha^2}{2}x^2}$
1	$\frac{3}{2}\hbar\omega$	$\frac{1}{\sqrt{2}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (2\alpha x)e^{-\frac{\alpha^2}{2}x^2}$
2	$\frac{5}{2}\hbar\omega$	$\frac{1}{\sqrt{8}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (4\alpha^2 x^2 - 2)e^{-\frac{\alpha^2}{2}x^2}$
3	$\frac{7}{2}\hbar\omega$	$\frac{1}{\sqrt{48}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (8\alpha^3 x^3 - 12\alpha x)e^{-\frac{\alpha^2}{2}x^2}$
4	$\frac{9}{2}\hbar\omega$	$\frac{1}{\sqrt{384}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (16\alpha^4 x^4 - 48\alpha^2 x^2 + 12)e^{-\frac{\alpha^2}{2}x^2}$
5	$\frac{11}{2}\hbar\omega$	$\frac{1}{\sqrt{3840}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (32\alpha^5 x^5 - 160\alpha^3 x^3 + 120\alpha x)e^{-\frac{\alpha^2}{2}x^2}$
6	$\frac{13}{2}\hbar\omega$	$\frac{1}{\sqrt{46080}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (64\alpha^6 x^6 - 480\alpha^4 x^4 + 720\alpha^2 x^2 - 120)e^{-\frac{\alpha^2}{2}x^2}$
7	$\frac{15}{2}\hbar\omega$	$\frac{1}{\sqrt{645120}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (128\alpha^7 x^7 - 1344\alpha^5 x^5 + 3360\alpha^3 x^3 - 1680\alpha x)e^{-\frac{\alpha^2}{2}x^2}$
8	$\frac{17}{2}\hbar\omega$	$\frac{1}{\sqrt{10321920}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (256\alpha^8 x^8 - 3584\alpha^6 x^6 + 13440\alpha^4 x^4 - 13440\alpha^2 x^2 + 1680)e^{-\frac{\alpha^2}{2}x^2}$
9	$\frac{19}{2}\hbar\omega$	$\frac{1}{\sqrt{185794560}}\left(\frac{\alpha^2}{\pi}\right)^{1/4} (512\alpha^9 x^9 - 9216\alpha^7 x^7 + 48384\alpha^5 x^5 - 80640\alpha^3 x^3 + 30240\alpha x)e^{-\frac{\alpha^2}{2}x^2}$