

BASIC EQUATIONS OF ELECTRODYNAMICS

Maxwell's Equations

In General:

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho \\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \end{cases}$$

In Matter:

$$\begin{cases} \vec{\nabla} \cdot \vec{D} = \rho_f \\ \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \end{cases}$$

Auxiliary Fields

Definitions:

$$\begin{cases} \vec{D} = \epsilon_0 \vec{E} + \vec{P} \\ \vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M} \end{cases}$$

Linear Media:

$$\begin{cases} \vec{P} = \epsilon_0 \chi_e \vec{E}, \quad \vec{D} = \epsilon \vec{E} \\ \vec{M} = \chi_m \vec{H}, \quad \vec{H} = \frac{1}{\mu} \vec{B} \end{cases}$$

Potentials

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

Lorentz Force Law

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Energy

$$U = \frac{1}{2} \int \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d\tau$$

FUNDAMENTAL CONSTANTS

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

(permittivity of free space)

$$\mu_0 = 4\pi \times 10^{-7} \text{ N} / \text{A}^2$$

(permeability of free space)

$$c = 3.00 \times 10^8 \text{ m/s}$$

(speed of light)

$$e = 1.60 \times 10^{-19} \text{ C}$$

(charge of the electron)

$$m = 9.11 \times 10^{-31} \text{ kg}$$

(mass of the electron)

SPHERICAL AND CYLINDRICAL COORDINATES

Spherical

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$

$$\begin{cases} \hat{x} = \sin \theta \cos \phi \hat{r} + \cos \theta \cos \phi \hat{\theta} - \sin \phi \hat{\phi} \\ \hat{y} = \sin \theta \sin \phi \hat{r} + \cos \theta \sin \phi \hat{\theta} + \cos \phi \hat{\phi} \\ \hat{z} = \cos \theta \hat{r} - \sin \theta \hat{\theta} \end{cases}$$

Cylindrical

$$\begin{cases} x = s \cos \phi \\ y = s \sin \phi \\ z = z \end{cases}$$

$$\begin{cases} \hat{x} = \cos \phi \hat{s} - \sin \phi \hat{\phi} \\ \hat{y} = \sin \phi \hat{s} + \cos \phi \hat{\phi} \\ \hat{z} = \hat{z} \end{cases}$$

FUNDAMENTAL THEOREMS

$$\text{Gradient Theorem: } \int_a^b (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

$$\text{Divergence Theorem: } \int (\nabla \cdot \vec{A}) d\tau = \oint \vec{A} \cdot d\vec{a}$$

$$\text{Curl Theorem: } \int (\vec{\nabla} \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

VECTOR DERIVATIVES

Cartesian. $d\vec{l} = dx \hat{x} + dy \hat{y} + dz \hat{z}; \quad d\tau = dx dy dz$

$$\text{Gradient: } \bar{\nabla}t = \frac{\partial t}{\partial x} \hat{x} + \frac{\partial t}{\partial y} \hat{y} + \frac{\partial t}{\partial z} \hat{z}$$

$$\text{Divergence: } \bar{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\text{Curl: } \bar{\nabla} \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \hat{z}$$

$$\text{Laplacian: } \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical. $d\vec{l} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}; \quad d\tau = r^2 \sin \theta dr d\theta d\phi$

$$\text{Gradient: } \bar{\nabla}t = \frac{\partial t}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\phi}$$

$$\text{Divergence: } \bar{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta v_\theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\begin{aligned} \text{Curl: } \bar{\nabla} \times \vec{v} = & \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{r} \\ & + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\theta) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi} \end{aligned}$$

Laplacian:

$$\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical. $d\vec{l} = ds \hat{s} + s d\phi \hat{\phi} + dz \hat{z}; \quad d\tau = s ds d\phi dz$

$$\text{Gradient: } \bar{\nabla}t = \frac{\partial t}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial t}{\partial \phi} \hat{\phi} + \frac{\partial t}{\partial z} \hat{z}$$

$$\text{Divergence: } \bar{\nabla} \cdot \vec{v} = \frac{1}{s} \frac{\partial}{\partial s} (sv_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\text{Curl: } \bar{\nabla} \times \vec{v} = \left[\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[\frac{\partial}{\partial s} (sv_\phi) - \frac{\partial v_s}{\partial \phi} \right] \hat{z}$$

$$\text{Laplacian: } \nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

VECTOR IDENTITIES

Triple Products

$$(1) \quad \bar{\mathbf{A}} \cdot (\bar{\mathbf{B}} \times \bar{\mathbf{C}}) = \bar{\mathbf{B}} \cdot (\bar{\mathbf{C}} \times \bar{\mathbf{A}}) = \bar{\mathbf{C}} \cdot (\bar{\mathbf{A}} \times \bar{\mathbf{B}})$$

$$(2) \quad \bar{\mathbf{A}} \times (\bar{\mathbf{B}} \times \bar{\mathbf{C}}) = \bar{\mathbf{B}} (\bar{\mathbf{A}} \cdot \bar{\mathbf{C}}) - \bar{\mathbf{C}} (\bar{\mathbf{A}} \cdot \bar{\mathbf{B}})$$

Product Rules

$$(3) \quad \bar{\nabla}(fg) = f(\bar{\nabla}g) + g(\bar{\nabla}f)$$

$$(4) \quad \bar{\nabla}(\bar{\mathbf{A}} \cdot \bar{\mathbf{B}}) = \bar{\mathbf{A}} \times (\bar{\nabla} \times \bar{\mathbf{B}}) + \bar{\mathbf{B}} \times (\bar{\nabla} \times \bar{\mathbf{A}}) + (\bar{\mathbf{A}} \cdot \bar{\nabla}) \bar{\mathbf{B}} + (\bar{\mathbf{B}} \cdot \bar{\nabla}) \bar{\mathbf{A}}$$

$$(5) \quad \bar{\nabla} \cdot (f \bar{\mathbf{A}}) = f(\bar{\nabla} \cdot \bar{\mathbf{A}}) + \bar{\mathbf{A}} \cdot (\bar{\nabla} f)$$

$$(6) \quad \bar{\nabla} \cdot (\bar{\mathbf{A}} \times \bar{\mathbf{B}}) = \bar{\mathbf{B}} \cdot (\bar{\nabla} \times \bar{\mathbf{A}}) - \bar{\mathbf{A}} \cdot (\bar{\nabla} \times \bar{\mathbf{B}})$$

$$(7) \quad \bar{\nabla} \times (f \bar{\mathbf{A}}) = f(\bar{\nabla} \times \bar{\mathbf{A}}) - \bar{\mathbf{A}} \times (\bar{\nabla} f)$$

$$(8) \quad \bar{\nabla} \times (\bar{\mathbf{A}} \times \bar{\mathbf{B}}) = (\bar{\mathbf{B}} \cdot \bar{\nabla}) \bar{\mathbf{A}} - (\bar{\mathbf{A}} \cdot \bar{\nabla}) \bar{\mathbf{B}} + \bar{\mathbf{A}} (\bar{\nabla} \cdot \bar{\mathbf{B}}) - \bar{\mathbf{B}} (\bar{\nabla} \cdot \bar{\mathbf{A}})$$

Second Derivatives

$$(9) \quad \bar{\nabla} \cdot (\bar{\nabla} \times \bar{\mathbf{A}}) = 0$$

$$(10) \quad \bar{\nabla} \times (\bar{\nabla} f) = 0$$

$$(11) \quad \bar{\nabla} \times (\bar{\nabla} \times \bar{\mathbf{A}}) = \bar{\nabla}(\bar{\nabla} \cdot \bar{\mathbf{A}}) - \nabla^2 \bar{\mathbf{A}}$$