

## Trigonometric Identities

$$\begin{aligned}\sin(\theta \pm \phi) &= \sin \theta \cos \phi \pm \cos \theta \sin \phi & \cos(\theta \pm \phi) &= \cos \theta \cos \phi \mp \sin \theta \sin \phi \\ \cos \theta \cos \phi &= \frac{1}{2} [\cos(\theta + \phi) + \cos(\theta - \phi)] & \sin \theta \sin \phi &= \frac{1}{2} [\cos(\theta - \phi) - \cos(\theta + \phi)] \\ \sin \theta \cos \phi &= \frac{1}{2} [\sin(\theta + \phi) + \sin(\theta - \phi)] & & \\ \cos^2 \theta &= \frac{1}{2} [1 + \cos 2\theta] & \sin^2 \theta &= \frac{1}{2} [1 - \cos 2\theta] \\ \cos \theta + \cos \phi &= 2 \cos \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2} & \cos \theta - \cos \phi &= 2 \sin \frac{\theta + \phi}{2} \sin \frac{\phi - \theta}{2} \\ \sin \theta \pm \sin \phi &= 2 \sin \frac{\theta \mp \phi}{2} \cos \frac{\theta \mp \phi}{2} & & \\ \cos^2 \theta + \sin^2 \theta &= 1 & \sec^2 \theta - \tan^2 \theta &= 1 \\ e^{i\theta} &= \cos \theta + i \sin \theta & & [\text{Euler's relation}] \\ \cos \theta &= \frac{1}{2} (e^{i\theta} + e^{-i\theta}) & \sin \theta &= \frac{1}{2i} (e^{i\theta} - e^{-i\theta})\end{aligned}$$

## Hyperbolic Functions

$$\begin{aligned}\cosh z &= \frac{1}{2} (e^z + e^{-z}) = \cos(iz) & \sinh z &= \frac{1}{2} (e^z - e^{-z}) = -i \sin(iz) \\ \tanh z &= \frac{\sinh z}{\cosh z} & \operatorname{sech} z &= \frac{1}{\cosh z} \\ \cosh^2 z - \sinh^2 z &= 1 & \operatorname{sech}^2 z + \tanh^2 z &= 1\end{aligned}$$

## Series Expansions

$$\begin{aligned}f(z) &= f(a) + f'(a)(z-a) + \frac{1}{2!} f''(a)(z-a)^2 + \frac{1}{3!} f'''(a)(z-a)^3 + \dots \quad [\text{Taylor series}] \\ e^z &= 1 + z + \frac{1}{2!} z^2 + \frac{1}{3!} z^3 + \dots & \ln(1+z) &= z - \frac{1}{2} z^2 + \frac{1}{3} z^3 - \dots \quad [|z| < 1] \\ \cos z &= 1 - \frac{1}{2!} z^2 + \frac{1}{4!} z^4 - \dots & \sin z &= z - \frac{1}{3!} z^3 + \frac{1}{5!} z^5 - \dots \\ \cosh z &= 1 + \frac{1}{2!} z^2 + \frac{1}{4!} z^4 + \dots & \sinh z &= z + \frac{1}{3!} z^3 + \frac{1}{5!} z^5 + \dots \\ \tan z &= z + \frac{1}{3} z^3 + \frac{2}{15} z^5 + \dots \quad [|z| < \pi/2] & \tanh z &= z - \frac{1}{3} z^3 + \frac{2}{15} z^5 - \dots \quad [|z| < \pi/2] \\ (1+z)^n &= 1 + nz + \frac{n(n-1)}{2!} z^2 + \dots \quad [|z| < 1] & & \quad [\text{binomial series}]\end{aligned}$$

## Some Derivatives

$$\begin{aligned}\frac{d}{dz} \tan z &= \sec^2 z & \frac{d}{dz} \tanh z &= \operatorname{sech}^2 z \\ \frac{d}{dz} \sinh z &= \cosh z & \frac{d}{dz} \cosh z &= \sinh z\end{aligned}$$

## Some Integrals

$$\begin{aligned}\int \frac{dx}{1+x^2} &= \arctan x & \int \frac{dx}{1-x^2} &= \operatorname{arctanh} x \\ \int \frac{dx}{\sqrt{1-x^2}} &= \arcsin x & \int \frac{dx}{\sqrt{1+x^2}} &= \operatorname{arcsinh} x \\ \int \tan x \, dx &= -\ln |\cos x| & \int \tanh x \, dx &= \ln |\cosh x| \\ \int \frac{dx}{x+x^2} &= \ln \left( \frac{x}{1+x} \right) & \int \frac{x \, dx}{1+x^2} &= \frac{1}{2} \ln(1+x^2) \\ \int \frac{dx}{\sqrt{x^2-1}} &= \operatorname{arccosh} x & \int \frac{x \, dx}{\sqrt{1+x^2}} &= \sqrt{1+x^2} \\ \int \frac{dx}{x\sqrt{x^2-1}} &= \arccos \left( \frac{1}{x} \right) & \int \frac{\sqrt{x} \, dx}{\sqrt{1-x}} &= \arcsin(\sqrt{x}) - \sqrt{x(1-x)} \\ \int \frac{dx}{(1+x^2)^{3/2}} &= \frac{x}{(1+x^2)^{1/2}} & \int \ln(x) \, dx &= x \ln(x) - x \\ \int_0^1 \frac{dx}{\sqrt{1-x^2} \sqrt{1-mx^2}} &= K(m), \quad \text{complete elliptic integral of first kind}\end{aligned}$$