## Spherical Coordinate System ( $r, \theta, \phi$ )



$$
\begin{aligned}
& x=r \sin \theta \cos \phi \\
& y=r \sin \theta \sin \phi \\
& z=r \cos \theta
\end{aligned}
$$

$$
\begin{aligned}
& \hat{x}=\sin \theta \cos \phi \hat{r}+\cos \theta \cos \phi \hat{\theta}-\sin \phi \hat{\phi} \\
& \hat{y}=\sin \theta \sin \phi \hat{r}+\cos \theta \sin \phi \hat{\theta}+\cos \phi \hat{\phi} \\
& \hat{z}=\cos \theta \hat{r}-\sin \theta \hat{\theta}
\end{aligned}
$$

Line element: $\quad d \mathbf{l}=d r \hat{r}+r d \theta \hat{\theta}+r \sin \theta d \phi \hat{\phi}$
Volume element: $d \tau=r^{2} \sin \theta d \theta d \phi d r$
Area element on surface of sphere ( $r=$ constant): $d a=r^{2} \sin \theta d \theta d \phi=r^{2} d \Omega$
Solid angle element: $d \Omega=\sin \theta d \theta d \phi$

Note: The definitions for $\theta$ and $\phi$ used here are those used in physics and engineering. A common convention in math textbooks has these definitions of $\theta$ and $\phi$ interchanged. (See $\underline{\mathrm{http}}: / /$ mathworld.wolfram.com/SphericalCoordinates.html for the prototype figure based on the common math convention for $\theta$ and $\phi$.)

# Cylindrical Coordinate System (s, $\phi, z$ ) 



$$
\begin{array}{ll}
x=s \cos \phi & \hat{x}=\cos \phi \hat{s}-\sin \phi \hat{\phi} \\
y=s \sin \phi & \hat{y}=\sin \phi \hat{s}+\cos \phi \hat{\phi}
\end{array}
$$

Line element: $\quad d \mathbf{l}=d s \hat{s}+s d \phi \hat{\phi}+d z \hat{z}$
Volume element: $d \tau=s d \phi d s d z$
Area element on cylindrical surface ( $s=$ constant): $\quad d a=s d \phi d z$
Area element on circular-disk surface ( $z=$ constant): $d a=s d \phi d s$

Note: The choice of the symbol $s$ for the radial coordinate, as used here and in Griffiths' textbook, is not the most common one. The symbols $\rho$ or $r$ are more commonly used in place of $s$. (Since the symbols $\rho$ and $r$ are used for other quantities in the Griffiths textbook, Griffiths uses $s$ to avoid confusion.)

