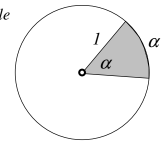
GEOMETRY OF RADIATION

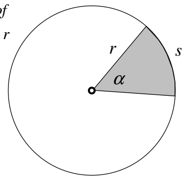
Plane Angle

unit circle



$\alpha = \text{arc length} \; [\text{radian}]$

circle of radius r



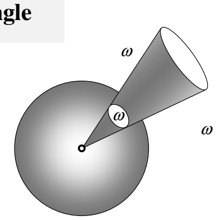
$$\frac{\alpha}{2\pi \cdot 1} = \frac{s}{2\pi \cdot r}$$

⇓

$$\boxed{\alpha = \frac{s}{r}}$$

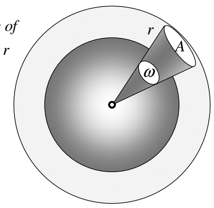
Solid Angle

unit sphere



$\omega = \text{area on a unit sphere}$ $[\text{steradian}], \; [sr]$

sphere of radius r

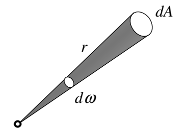


$$\frac{\omega}{4\pi \cdot 1^2} = \frac{A}{4\pi \cdot r^2}$$

⇓

$$\boxed{\omega = \frac{A}{r^2}}$$

Differential Solid Angle



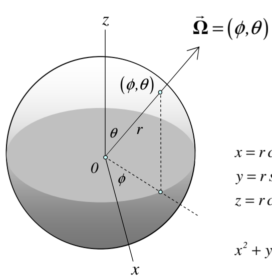
$$\boxed{d\omega = \frac{dA}{r^2}}$$

Differential Solid Angle in spherical coordinates

Direction is defined by a pair of angles: $\vec{\Omega} = (\phi, \theta)$

ϕ *is an azimutal angle:* $0 \le \phi \le 2\pi$

θ *is a polar angle:* $0 \le \theta \le \pi$



$x = r\cos\phi\sin\theta$

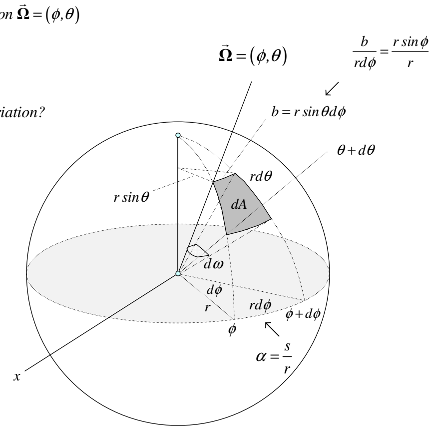
$y = r\sin\phi\sin\theta$

$z = r\cos\theta$

$x^2 + y^2 + z^2 = r^2$

Consider a differential variation of the direction $\vec{\Omega} = (\phi, \theta)$ $\phi + d\phi$ *and* $\theta + d\theta$

What solid anlgle corresponds to this variation?



$$dA \approx (r\sin\theta d\phi) \cdot (r d\theta) = r^2 \sin\theta d\phi d\theta$$

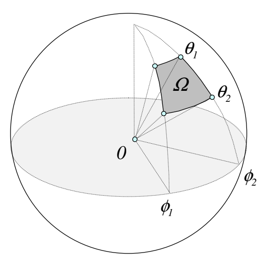
differential solid angle

$$\boxed{d\omega = \frac{dA}{r^2} = \sin\theta d\phi d\theta} \qquad (12.3)$$

Finite Solid Angle in spherical coordinates

Consider a finite solid angle bounded by the directions:

$\phi_1 \le \phi \le \phi_2$ *and* $\theta_1 \le \theta \le \theta_2$



solid angle $$\Omega = \iint_{\Omega} d\omega = \int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} \sin\theta d\phi d\theta$$

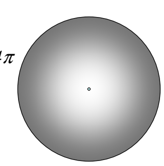
$$= -\int_{\phi_1}^{\phi_2} \int_{\cos(\theta_1)}^{\cos(\theta_2)} d\phi d\mu$$

$\mu = \cos\theta$ (*directional cosine*)

$$= (\phi_2 - \phi_1)(\cos\theta_1 - \cos\theta_2)$$

Sphere

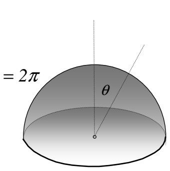
$\Omega_{sphere} = 4\pi$



$$\Omega_{sphere} = \Omega_{\bigcirc} = \int_0^{\pi} \int_0^{2\pi} \sin\theta d\phi d\theta = \boxed{4\pi}$$

Hemisphere

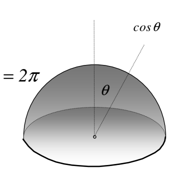
$\Omega_{hemisphere} = 2\pi$



$$\Omega_{hemisphere} = \int_0^{\pi/2} \int_0^{2\pi} \sin\theta d\phi d\theta = \boxed{2\pi}$$

Useful Fact

$\Omega_{hemisphere} = 2\pi$



$$\int_{2\pi} \cos\theta d\omega = \int_0^{\pi/2} \int_0^{2\pi} \cos\theta \sin\theta d\phi d\theta = \boxed{\pi}$$