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Unit 2.7
Our data should come out with a **bell-shaped** “curve”

⇒ Normal distribution
⇒ Gaussian distribution

If we were to find an equation that models our histogram, it would be:

\[
f(t) = \frac{1}{\sigma_{sd} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{(t-\langle t \rangle)^2}{\sigma_{sd}^2} \right)}
\]
Average and Standard Deviation ($\sigma_{sd}$)

Last time, we found that for one data set with multiple measurements:

⇒ Best estimate of the true value = Average: $<t>$ or $<x>$, etc.
⇒ A measure of the spread of values = Standard Deviation: $\sigma_{sd}$
**Average and Standard Deviation** ($\sigma_{sd}$)

Last time, we found that for one data set with multiple measurements:

$\Rightarrow$ Best estimate of the true value = **Average**: $< t >$ or $< x >$, etc.

$\Rightarrow$ A measure of the spread of values = **Standard Deviation**: $\sigma_{sd}$

Edward’s results for the ball drop were: $< \Delta t > = 0.421 \text{ s} \quad \sigma_{sd} = 0.035 \text{ s}$
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Linda’s results for the ball drop were: $< \Delta t > = 0.653 \text{ s} \quad \sigma_{sd} = 0.077 \text{ s}$
**Average and Standard Deviation** \((\sigma_{sd})\)

Last time, we found that for one data set with multiple measurements:

⇒ Best estimate of the true value = Average: \( < t > \) or \( < x > \), etc.
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Linda’s results for the ball drop were:  $< \Delta t > = 0.653$ s  $\sigma_{sd} = 0.077$ s
Standard Deviation of the Mean (SDM)

But $\sigma_{sd}$ does not really tell us the uncertainty of the average, it just tells us the spread of our data measurements. We really need to use:

$\Rightarrow$  Standard deviation of the mean (SDM) (Standard error)
Standard Deviation of the Mean (SDM)

But $\sigma_{sd}$ does not really tell us the uncertainty of the average, it just tells us the spread of our data measurements. We really need to use:

$\Rightarrow$ Standard deviation of the mean (SDM) (Standard error)

$$SDM \equiv \frac{\sigma_{sd}}{\sqrt{N}}$$

$\Rightarrow$ 68.3% chance that another data set of multiple measurements will have an average that is within average $\pm SDM$
Edward:

\[ \langle \Delta t \rangle = 0.421 \text{ s} \quad \sigma_{sd} = 0.035 \text{ s} \quad SDM \equiv \frac{\sigma_{sd}}{\sqrt{N}} = \frac{0.035 \text{ s}}{\sqrt{20}} = 0.0078 \text{ s} \]

\[ \Rightarrow \quad \langle \Delta t \rangle = 0.421 \text{ s} \pm 0.008 \text{ s} \]
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Linda:

\[ \langle \Delta t \rangle = 0.653 \text{ s} \quad \sigma_{sd} = 0.077 \text{ s} \quad SDM = \frac{\sigma_{sd}}{\sqrt{N}} = \frac{0.077 \text{ s}}{\sqrt{20}} = 0.0172 \text{ s} \]

\[ \Rightarrow \quad \langle \Delta t \rangle = 0.65 \text{ s} \pm 0.02 \text{ s} \]
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\[\Rightarrow \quad < \Delta t > = 0.65 \text{ s} \pm 0.02 \text{ s} \]
Unit 2.8

We need to solve an equation to find the true time for an object to fall 2.0 m, starting from rest.

The actual equation (which we will develop experimentally) is given by:

\[ y_2 - y_1 = \frac{1}{2}a_y(t_2 - t_1)^2 + v_1(t_2 - t_1) \]

But

1) Ball starts at rest \( \Rightarrow \) \( v_1 = 0.0 \text{ m/s} \)
2) We can choose \( t_1 = 0.0 \text{ s} \) and \( y_1 = 0.0 \text{ m} \) (or \( y_2 = 0.0 \text{ m} \))

So

\[ y_2 = \frac{1}{2}a_yt_2^2 \]

- Solve all equations symbolically first.
- Substitute in numbers with units at the very end.
- Pay careful attention to the signs – they have physical meaning.
- Remember that the square root of a number has 2 answers.
First, multiply both sides by 2  \[ 2y_2 = 2 \frac{1}{2} a_y t_2^2 \]

Divide both sides by \( a_y \)  \[ \frac{2y_2}{a_y} = \frac{a_y t_2^2}{a_y} \]

Take the square root of both sides  \[ \sqrt{\frac{2y_2}{a_y}} = \sqrt{t_2^2} \]

Rearrange, with unknown on left side  \[ t_2 = \sqrt{\frac{2y_2}{a_y}} \]

Substitute in numbers (with proper units and signs):

\[
t_2 = \sqrt{\frac{2y_2}{a_y}} = \sqrt{\frac{2(+2.0 \text{ m})}{9.8 \text{ m/s/s}}} = \pm 0.639 \text{ s}
\]
True time for a ball to fall 2.0 m (from rest): $\Delta t = t_2 - t_1 = + 0.639$ s
True time for a ball to fall 2.0 m (from rest): $\Delta t = t_2 - t_1 = + 0.639 \text{ s}$
True time for a ball to fall 2.0 m \textit{(from rest)}: \( \Delta t = t_2 - t_1 = +0.639 \text{ s} \)
True time for a ball to fall 2.0 m \textbf{(from rest)}: \( \Delta t = t_2 - t_1 = +0.639 \text{ s} \)

For the majority of you, your range of \( <\Delta t> \pm \text{SDM} \) does \textbf{not} include the theoretical value of 0.639 s.

\[ \Rightarrow \quad \text{Systematic error} \]

Sources of the error: Ask yourself

\begin{itemize}
  \item what quantities did I measure?
  \item what did I “use” to measure them?
  \item how did I measure them?
\end{itemize}

(Same questions are asked to find sources of uncertainty)
For the ball drop:

- **Distance**
  - Meter stick: Check against a standard. etc.
  - Human technique: Check if you are using stick properly. Placing the ball at the correct height. etc.

- **Time**
  - Stopwatch: Check if it is functioning properly. etc.
  - Human technique: Proper starting and stopping. Giving the ball an initial speed. etc.