f. As you will see in the next activity, the value of c_2 should be about half of the cart acceleration. Is it?

THE KINEMATIC EQUATIONS

4.8. 1D KINEMATIC EQUATIONS FOR CONSTANT ACCELERATION

So far, you should have concluded that some of the motions have had accelerations that are more or less constant. *There is a standard set of equations (which can be derived using the principles of calculus) that describe the motion of an object that undergoes constant acceleration*. These equations are called the kinematic equations and they are derived in most standard physics textbooks. By re-examining the graphs you have sketched that describe objects moving with constant acceleration, and by using the definitions of instantaneous velocity and acceleration, you can verify that the kinematic equations describe uniformly accelerated motion. We will use the following symbols for this exercise:

x_2 = position along the x-axis (which can vary with time)

v_{2x} = instantaneous velocity along the x-axis (which can also vary with time) at some later time t_2.

a_x = represents an acceleration along the x-axis. (In this special situation, it does not vary in time because we have chosen to consider only those motions for which a_x is constant.)

t_1 = time at which motion of interest starts

x_1 = initial position at t_1

v_{1x} = initial velocity component along the x-axis at t_1

$(t_2 - t_1)$ = the time elapsed since the object was at x_1.

Beware: The kinematic equations you are about to derive in the space below *only apply when an object undergoes constant acceleration*.

There are four constant acceleration kinematic equations commonly found in physics textbooks. The most fundamental kinematic equation is the equation describing x_2 as a function of t_2 when the initial position is x_1 and the initial velocity is v_{1x}.

Kinematic Equation #1:

$$x_2 - x_1 = v_{1x}(t_2 - t_1) + \frac{1}{2}a_x(t_2 - t_1)^2 \quad (\text{for } a_x = \text{constant}) \tag{4.2}$$

This equation indicates that a graph showing the position as a function of time of any motion with constant acceleration is a parabola of some sort. In fact, you should have verified the experimental validity of this equation in the mathematical modeling exercise you did in Activity 4.7.1. In that activity you should have determined that a graph of a parabola like that shown in Equation 4.2 and a graph of your data are almost identical. **Note:** All other kinematic equations can be obtained from the fundamental equation and the definitions of instantaneous velocity and acceleration.

of problem, an example will be presented and worked, and then you will be asked to work a practice problem.

TYPE ONE PROBLEMS

In type one problems we usually know the time an object is accelerating and some combination of other variables. In the case of type one problems Kinematic Equation #1 is the key equation.

Type One Kinematics Problems

Example Problem About a Runner:

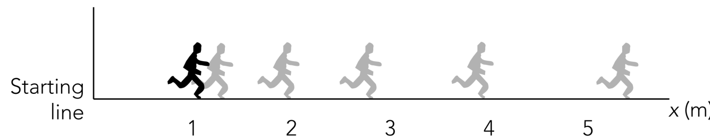

Fig. 4.15.

Carl Lewis is hoping for a record finishing time in the 100 meter run. During the first meter Lewis gets off the blocks into his upright running position and is already moving at a speed of 3.4mi/h. During the next four meters of his run he accelerates at a constant rate to his final running speed. Yolanda, a young sport scientist who is studying his acceleration, clocks the time that it took Lewis to run from 1.0m to 5.0m as 0.69s. What is his acceleration?

Practice Problem About a Jet Boat: The Rogue River in Oregon is so rocky in spots that jet boats have become a popular way for tourists to see wild stretches of the river. A system of air jets thrusting downward keep the boat suspended above the water and a second system of air jets move it horizontally. A jet boat driver comes around a bend in the river moving due west at a speed of 12m/s when she discovers that a large Douglas fir tree has fallen across the river 24meters in front of her. She quickly reverses her horizontal jets so that they deliver a constant acceleration in an easterly direction. The jet boat slows down considerably and reaches the log after 4.0 seconds. What is its acceleration? Does it stop in time?

On the pages that follow all four parts of a suggested solution to the runner problem are described. After each element is explained you will have a chance to practice using these elements to solve the jet boat problem in which a jet boat driver avoids a collision with a fir tree.

Part 1: Motion Diagrams and Graphs.

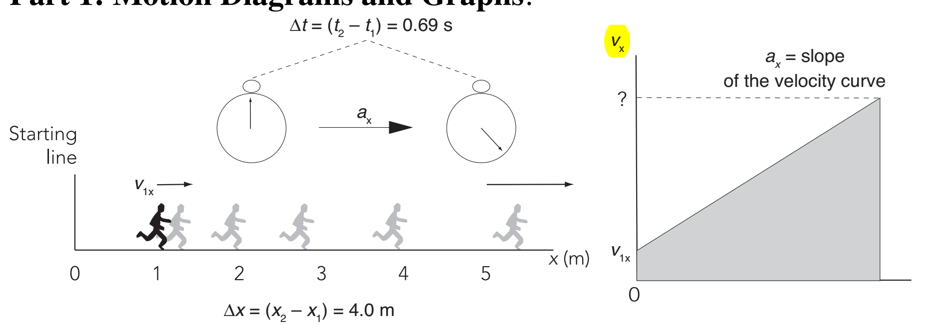

Fig. 4.16. A motion diagram and graph used for the formal solution of kinematics problems.

Notice that we depict the runner at his known positions at the beginning and end of the time interval. The arrows associated with his velocity are shown at those two locations and have different lengths–longer for greater velocity. The direction of the acceleration is shown with an arrow also. All the relevant variables are indicated by their symbols. The diagram is an abstract representation of the problem. A second representation is the sketched graph. The slope represents the acceleration, the area represents the displacement, and the initial velocity and the time interval are also shown. It is sometimes possible to solve the problem merely by constructing a graph.

4.9.1. Activity: Jet Boat Problem Motion Diagram and Graph

a. Draw a motion diagram to represent practice kinematics problem about the jet boat.

b. Sketch a velocity vs. time graph for the jet boat assuming it has a constant acceleration.

Part 2: Table of Values and Equations

List all the values given in the problem and convert to a common set of units you plan to use. This is usually based on the International System of Units which is abbreviated SI (based on the French version of the name). This system is sometimes referred to as the MKS system to reflect its use of meters, kilograms and seconds as fundamental units.

In this example the runner's initial and final position was given and the time required to move between those positions was given. Thus, you will need to use Kinematic Equation #1.

$$x_2 - x_1 = v_{1x}(t_2 - t_1) + \frac{1}{2}a_x(t_2 - t_1)^2$$

The table of values, conversions, and the derivation of this equation are shown as follows.

Part 2: Table and Unit Conversions

$$x_1 = 1.0\text{m}$$
$$x_2 = 5.0\text{m}$$
$$v_{1x} = 3.4\text{mi/h}$$
$$= \left(.447\frac{\text{m/s}}{\text{mi/h}}\right)\left(3.4\frac{\text{mi}}{\text{h}}\right)$$
$$= 1.52\text{m/s}$$
$$(1\text{mi/h} = 0.447\text{m/s})$$
$$t_2 - t_1 = 0.69\text{s}$$

Part 2: Runner Equations

Only one equation is needed for this situation but it must be solved for acceleration

$$x_2 - x_1 = v_{1x}(t_2 - t_1) + a_x(t_2 - t_1)^2$$

4.9.2. Activity: A Table and Equations for the Jet Boat

a. Make up a table of values to represent practice problem about the jet boat. Be sure to show any unit conversions, if needed.

b. Write down or derive the basic equations needed to find the acceleration of the jet boat as a function of the distance it moves and its initial position and velocity and the time the motion takes. Do not solve the equation(s) for acceleration quite yet.

Part 3: Runner Algebra and Substitution

The next step is to solve the equation algebraically for the quantity of interest. In this problem it is the acceleration along the x-axis, a_x, but in other problems the acceleration may be known and a velocity or the distance moved might be of more interest.

Runner Algebra and Substitution

Solve for a_x:

Since $x_2 - x_1 = v_{1x}(t_2 - t_1) + \frac{1}{2}a_x(t_2 - t_1)^2$, then $a_x = \frac{2\left[(x_2 - x_1) - v_{1x}(t_2 - t_1)\right]}{(t_2 - t_1)^2}$.

so $a_x = \frac{2\left[(5.0\,\text{m} - 1.0\,\text{m}) - (1.5\,\text{m/s})(0.69\,\text{s})\right]}{(0.69\,\text{s})^2} = 13\,\text{m/s}^2$

ANSWER (with proper significant figures): a = 13 m/s^2

4.9.3. Activity: Jet Boat Algebra and Substitution

a. Solve the equation obtained in Part 2 for the acceleration of the practice problem jet boat algebraically.

b. Substitute the values in appropriate units into the equation and calculate the acceleration.

Part 4: Units Check

The final step is to see that the units represented on the right hand side of your final equation used in Part 3 and the units on the left hand side are the *same*. For example, for the case of our runner:

Runner Units Check

$$[\text{m/s/s}] = \frac{[\text{m}] - [\text{m/s}][\text{s}]}{[\text{s}^2]} = \frac{[\text{m}]}{[\text{s}^2]}$$

4.9.4. Activity: Jet Boat Units Check

Check to see that the units on both sides of the final equation you are using to find the acceleration of the jet boat are the same.

TYPE TWO PROBLEMS

In type two problems we usually know both the initial and the final velocity of an object and either the acceleration of the object or the distance over which it moves. In the case of type two problems, you can either use a combination of Kinematic Equation #1 and the definition of acceleration or you can use Kinematic Equation #3 which will be derived later in this section.

Type 2 Kinematics Problems

Example Problem 2:

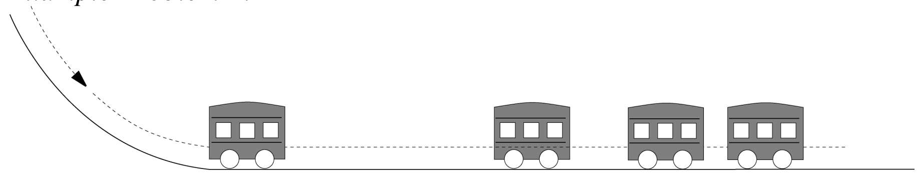

Fig. 4.17.

A cage holding four people at the Cedar Point Amusement Park has been accelerated as the result of a vertical free fall. It changed direction on a track and is coasting horizontally with an initial velocity of 40 m/s when the brakes are applied. The cart slows down at a constant acceleration and stops in a distance of 18 m. What is its acceleration due to the braking action?

Practice Problem 2: A Boeing 757 jet with 200 passengers and full fuel tanks has a constant acceleration at full throttle of 2.0 m/s/s and must be going 170 mi/h to lift off. You have to decide as a Federal Aviation Administration safety officer whether to allow United Airlines to operate the Boeing 757 on a new sea level airfield with a runway length of 1.00 miles. Will you allow United to operate B757s on the new airfield?

On the pages that follow all four parts of a solution to the example problem on the stopping cage are described. After each element is explained you will have a chance to practice using it to solve the practice problem in which a Boeing 757 jet takes off from an airfield that might be too short.

Part 1: Motion Diagrams and Graphs

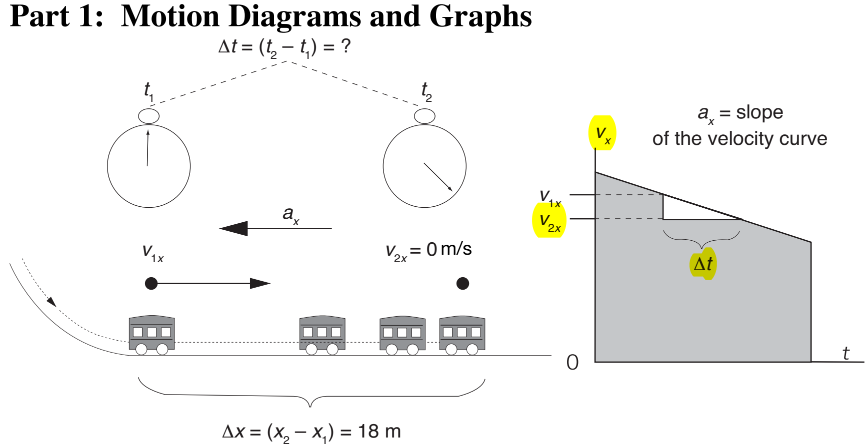

Fig. 4.18. A motion diagram and graph used for the formal solution of kinematics problems.

Notice that in Figure 4.18 we see the cart twice—once at the beginning of the time interval, in the state of motion described by v_{1x}, and once at the end of the time interval in the state of motion described by v_{2x}. The arrows associated with each object have different lengths—longer for greater velocity. All the relevant variables are indicated by their symbols. The diagram is, in effect, an abstract representation of the problem. A second representation is the sketch graph on the right. The slope represents the acceleration, the area represents the displacement, and both velocities and the time interval are also shown. It is sometimes possible to solve the problem merely by constructing a graph.

4.9.5. Activity: A Motion Diagram and Graph for the Jet Plane

a. Draw a motion diagram to represent practice problem 2 about the Boeing 757.

b. Sketch a velocity vs. time graph for the Boeing 757 assuming it is taking off with a constant acceleration.

Part 2: Table of Values and Equations

List all the values given in the problem and convert to a common set of units you plan to use. This is usually based on the SI system in which lengths are in meters and time in seconds.

In this example the initial and final velocity of the cage was given and the distance moved between them, $(x_2 - x_1)$ was given instead of the time. Thus, you will need to use Kinematic Equation #1 and its first derivative to eliminate time. Doing this gives a very important additional kinematic equation relating velocities, distance moved, and acceleration. This equation is the third kinematic equation.

Kinematic Equation #3:

$$(v_{2x})^2 = (v_{1x})^2 + 2a_x(x_2 - x_1) \qquad (4.4)$$

The table of values, conversions, and the derivation of this third kinematic equation are shown as follows.

Part 2: Table and Unit Conversions

Initial: $v_{1x} = 40$ m/s

Final: $v_{2x} = 0.0$ m/s

Distance Moved:

$$(x_2 - x_1) = 18\,\text{m}$$

$$a_x = ?$$

Part 2: Equations for the Stopping Cage

$$x_2 - x_1 = v_{1x}(t_2 - t_1) + \frac{1}{2}a_x(t_2 - t_1)^2$$

$$\text{and } v_{2x} = \frac{dx_2}{dt} = v_{1x} + a_x(t_2 - t_1),$$

$$\text{so } \Delta t = t_2 - t_1 = \frac{v_{2x} - v_{1x}}{a_x}, \text{ or}$$

$$x_2 - x_1 = v_{1x}\left(\frac{v_{2x} - v_{1x}}{a_x}\right) + \frac{1}{2}a_x\left(\frac{v_{2x} - v_{1x}}{a_x}\right)^2$$

$$= \frac{1}{a_x}\left((v_{2x})(v_{1x}) - (v_{1x})^2\right)$$

$$+\frac{1}{2a_x}\left((v_{2x})^2 - 2(v_{2x})(v_{1x})^2 + (v_{1x})^2\right)$$

$$x_2 - x_1 = \frac{1}{2a_x}\left((v_{2x})^2 - (v_{1x})^2\right)$$

so we get the third kinematic equation:

$$(v_{2x})^2 = (v_{1x})^2 + 2a_x(x_2 - x_1)$$

4.9.6. Activity: A Table and Equations for the Jet Plane

a. Make up a table of values to represent the practice problem about the Boeing 757. Be sure to show any unit conversions.

b. Write down or derive the basic equations needed to find the horizontal distance the Boeing 757 moves as a function of its acceleration and its initial and final velocities.

Part 3: Algebra and Substitution

The next step is to solve the equation algebraically, if needed, for the quantity of interest. In this problem it is the acceleration, a, but in other problems the acceleration may be known and a velocity or the distance moved might be of more interest.

Stopping Cage Algebra and Substitution

Solving Kinematic Equation for the Stopping Cage for acceleration gives

$$a_x = \frac{\left((v_{2x})^2 - (v_{1x})^2\right)}{2(x_2 - x_1)} = \frac{\left((0\,\mathrm{m/s})^2 - (40\,\mathrm{m/s})^2\right)}{2(18\mathrm{m})}$$

ANSWER (with proper significant figures) $a_x = -44\ \mathrm{m/s^2}$

4.9.7. Activity: Algebra and Substitution for the Jet Plane

a. Solve the equation obtained in Part 3 for the horizontal distance of the Boeing 757 as it moves before it reaches take off velocity.

b. Substitute the values in appropriate units into the equation and calculate the distance moved by the Boeing 757.

c. As an FAA safety officer would you allow United Airlines to operate Boeing 757s on the new airfield? Explain.

Part 4: Units Check

The final step is to see that the units represented on the right hand side of your final equation used in Part 3 and the units on the left hand side are the same. For example, for the case of our stopping cage:

Units Check for Stopping Cage

$$[\mathrm{m/s/s}] = \frac{[(\mathrm{m/s})^2]}{[\mathrm{m}]} = \frac{[\mathrm{m^2/s^2}]}{[\mathrm{m}]} = \frac{[\mathrm{m}]}{[\mathrm{s^2}]} = [\mathrm{m/s/s}]$$

4.9.8. Activity: Units Check for the Jet Plane

Check to see that the units on both sides of the equation agree.