7.1. OVERVIEW

In the last unit you began the study of the application of Newton's laws to *projectile motion*. In this unit we are going to consider the application of Newton's laws to several other phenomena in one and two dimensions. Since Newton's laws can be used to predict types of motion or the conditions for no motion, they are used in many endeavors including astrophysics, engineering, and the study of human body motion.

You will begin this unit by exploring *uniform circular motion,* in which an object moves at a constant speed in a circle. In particular, you will develop a mathematical description of centripetal acceleration and the force needed to keep a massive object moving in a circle.

Next you will consider the characteristics of several "invisible" forces that must be taken into account during the comprehensive application of Newton's laws to problems of real interest. These forces—*friction, tension in strings,* and *normal forces*—are called *passive forces* because they only act in response to other forces.

Finally, you will learn techniques for drawing free-body diagrams to help you predict motions or find the conditions for equilibrium (*i.e.*, no acceleration) in some relatively complicated situations. One of these situations includes the study of motions up and down ramps or inclined planes in which vector components of forces must be considered. In inclined plane motion, the path of the object is constrained to move along a straight line and is not parabolic like that of a projectile.

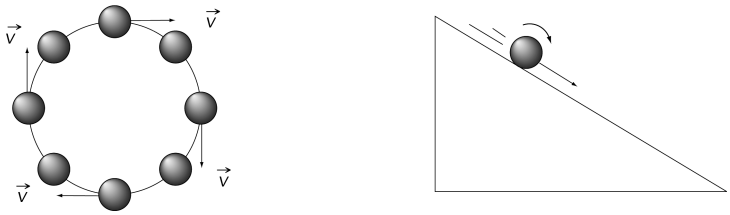


Fig. 7.1. Two types of motion to be explored in this unit: uniform circular motion with constant speed, and frictionless "falling" motion along a straight line that lies along the surface of a ramp.

7.4.1. Activity: How Does *a* Depend on *v* and *r*?

a. Do you expect the centripetal acceleration to be more, or less, for an object moving at a certain speed to rotate in a smaller circle? In other words, would the magnitude, a, increase or decrease as r decreases if circular motion is maintained at the same constant speed? Explain.

b. Do you expect the centripetal acceleration to be more, or less, for an object rotating at a given radius r if the speed v is increased? In other words, would the magnitude, a, increase or decrease as v increases if circular motion is maintained at the same constant radius? Explain.

You should have guessed that it requires more acceleration to move an object of a certain speed in a circle of smaller radius and that it also takes more acceleration to move an object that has a higher speed in a circle of a given radius. Let's use the definition of acceleration in two dimensions and some accepted mathematical relationships to show that the magnitude of centripetal acceleration, a, should actually be given by the equation

$$a = \frac{v^2}{r} \quad \text{(magnitude of centripetal acceleration)} \tag{7.1}$$

To do this derivation you will want to use the following definition for acceleration

$$\langle \vec{a} \rangle \equiv \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} \tag{7.2}$$

7.4.2. Activity: Finding the Equation for *a*

a. Refer to the diagram that follows. Show that, at the two points shown on the circle, the angle between the position vectors at times t_1 and t_2 is the same as the angle between the velocity vectors at times t_1 and t_2. **Hint:** In circular motion, velocity vectors are always perpendicular to their position vectors.

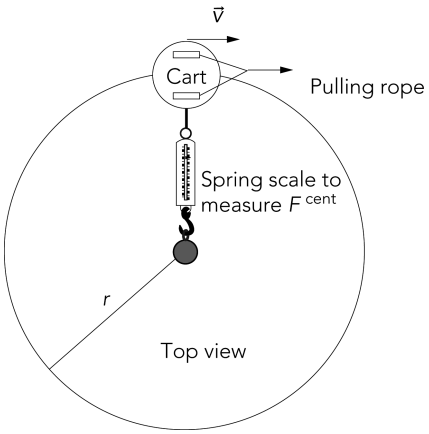


Fig. 7.5. Top view of centripetal force experiment.

7.5.1. Activity: Verifying the F^{cent} Equation

a. If a puller applies a force on the cart that is always tangent to the circle and just sufficient to balance friction in the cart and maintain the cart's motion at a constant speed, what is the *net* force in a direction perpendicular to the circle? Remember Newton's first and second laws!!??

b. You and members of your group should practice pulling each of your members around at a constant speed. The rider doesn't need the belt for this exercise. Instead, the rider should hold onto the rope and close his or her eyes and concentrate on feeling the centripetal force.

c. When you hold onto the center rope while riding on the cart, what is the direction of the *net* force you feel on you (include consideration of the forces in the radial direction and in the tangential direction)? Does the force you feel seem to increase as you rotate faster? As you rotate in a smaller circle? Explain.

d. Next, keep the radius to the center of the cart and the mass of the cart and rider constant and take data for several different orbital speeds with the rope attached to the belt that the rider is wearing. Set up a spreadsheet and compare the theoretical value of F^{cent} calculated using equation 7.3 with the experimental value obtained by reading the average force in newtons recorded on the spring scale. Use the space below to explain your procedures, show sample calculations, and present your results. **Note:** When reading the force scale you should