This observation relates a fixed torque applied by you to the resulting rotational velocity of a spinning rod with masses on it. When the resulting rotational acceleration is small for a given effort, we say that the rotational inertia is large. Conversely, a small rotational inertia will lead to a large rotational acceleration. In this observation you can place masses at different distances from an axis of rotation to determine what factors cause rotational inertia to increase.

Center a light aluminum rod on the almost frictionless pivot that is fixed at your table. With your finger, push the rod at a point about halfway between the pivot point and one end of the rod. Spin the rod gently with different mass configurations as shown in the figure below.

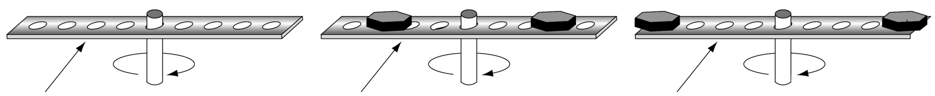

Fig. 12.13. Causing a rod to rotate under the influence of a constant applied torque for three different mass configurations.

12.9.1. Activity: Rotational Inertia Factors

a. What do you predict will happen if you exert a constant torque on the rotating rod using a uniform force applied by your finger at a fixed lever arm? Will it undergo a rotational acceleration, move at a constant rotational velocity, or what?

b. What do you expect to happen differently if you use the same torque on a rod with two masses added to the rod as shown in the middle of Figure 12.13?

c. Will the motion be different if you relocate the masses farther from the axis of rotation as shown in Figure 12.13 on the right? If so, how?

d. While applying a constant torque, observe the rotation of: (1) the rod, (2) the rod with masses placed close to the axis of rotation, and (3) the rod with the same masses placed far from the axis of rotation. Look carefully at the motions. Does the rod appear to undergo rotational acceleration or does it move at a constant rotational velocity?

e. What is the equation for the rotational inertia, I, of a hoop of radius r and mass M rotating about its center?

The Rotational Inertia of a Disk

The basic equation for the rotational inertia of a point mass is mr^2. Note that as r increases I increases, rather dramatically, as the square of r. Let's consider the rolling motion of a hoop and disk both having the same mass and radius. To make the observation of rotational motion your class will need one setup of the following demonstration apparatus:

- 1 hoop
- 1 disk
- 1 ramp

Recommended Group Size:	All	Interactive Demo OK?:	Y

12.11.2. Activity: Which Rotational Inertia Is Larger?

a. If a hoop and a disk both have the same outer radius and mass, which one will have the larger rotational inertia? **Hint:** Which object has its mass distributed farther away from an axis of rotation through its center? Why?

b. Which object should be more resistant to rotation—the hoop or the disk? Explain. **Hint:** You may want to use the results of your observation in Activity 12.9.1c.

c. What will happen if a hoop and disk each having the same mass and outer radius are rolled down an incline? Which will roll faster? Why?

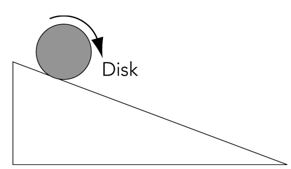

Fig. 12.15. A disk rolling down an incline.

d. What did you actually observe, and how valid was your prediction?

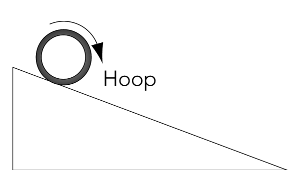

Fig. 12.16. A hoop rolling down an incline.

It can be shown experimentally that the rotational inertia of any rotating body is the sum of the rotational inertias of each tiny mass element, *dm,* of the rotating body. If an infinitesimal element of mass, *dm,* is located at a distance r from an axis of rotation, then its contribution to the rotational inertia of the body is given by r^2dm. Mathematical theory tells us that since the total rotational inertia of the system is the sum of the rotational inertia of each of its mass elements, the rotational inertia I is the integral of r^2dm over all m. This is shown in the equation below.

$$I = \int r^2\,dm$$

When this integration is performed for a disk or cylinder rotating about its axis, the rotational inertia turns out to be

$$I = \frac{1}{2}MR^2$$

where M is the total mass of the cylinder and R is its radius. See almost any standard introductory physics textbook for details of how to do this integral.

A disk or cylinder can be thought of as a series of nested, concentric hoops. This is shown in the following figure.

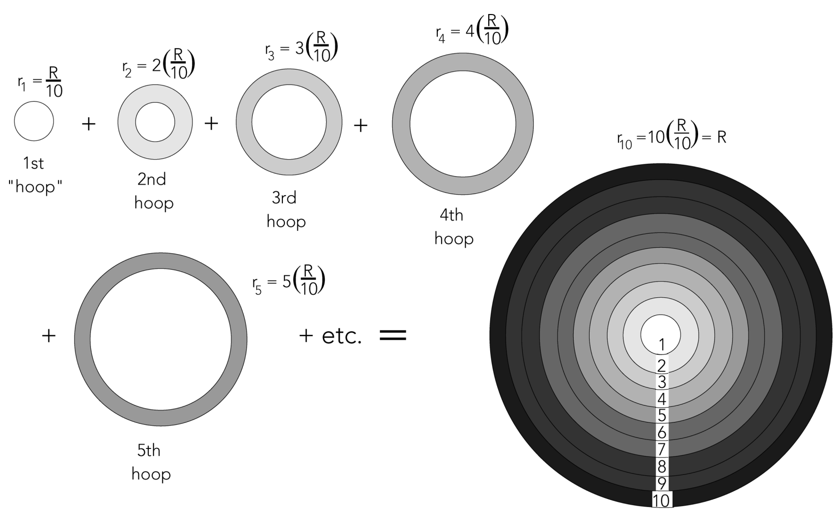

Fig. 12.17. A disk or cylinder constructed from a set of ten concentric hoops.

It is instructive to compare the theoretical rotational inertia of a disk, calculated using integral calculus, with a spreadsheet calculation of the rotational inertia approximated as a series of ten concentric hoops each of width R/n.

Suppose the disk pictured in Figure 12.17 is a life-sized drawing of a disk that has a total mass, M, of 2.0 kg. Assume that the disk has a uniform density and a constant thickness.

12.11.3. Activity: The Rotational Inertia of a Disk

a. Measure the radius, R, of the disk shown in Figure 12.17. Use the theoretical equation obtained from integration to calculate the theoretical value of the rotational inertia of that disk. The theoretical equation is

$$I = \int r^2\, dm = \frac{1}{2} MR^2$$

b. You will need to find an equation for the area of a hoop or radius, r, in order to determine what fraction of the total mass of the disk is contained in each hoop. If the area of a disk of radius, r, is given by $A = \pi r^2$, show that the area of the first inner disk, is given by $A_1 = \pi r_1^2$. Since there are 10 hoops of equal "width," $r_1 = R/10$.

c. Show that the area of the second hoop can be calculated by subtracting the area of the inner disk from the area of the second disk so that $A_2 = \pi r_2^2 - \pi r_1^2$ where $r_2 = 2(R/10)$.

d. Show that the area of the nth hoop is given by subtracting the area of the $(n - 1)$th disk from the nth disk so that $A_n = \pi r_n^2 - \pi r_{n-1}^2$ where $r_n = n(R/10)$, and $r_{n-1} = (n - 1)(R/10)$.

e. Create a spreadsheet with the values of n, r_n, and A_n for each hoop along with the value of rotational inertia contributed by each of the hoops. Affix the spreadsheet in the space following. **Hint:** If the disk has a uniform density, then the mass of each hoop, m_n, is proportional to its area, A_n, so that $m_n = (A_n/A_{\text{disk}})M$ where $A_{\text{disk}} = \pi R^2$.