How To Scientific Notation

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What Is Scientific Notation And How Is It Used?

Scientific notation is also referred to as exponential notation. It is based on the *Law of Exponents*. The notation is based on powers of base number 10. The *Standard Notational Form* is:

$N \times 10^x$ where N is a number greater than 1 but less than 10 and x is an integer exponent of 10.

Placing numbers in exponential notation has several advantages.

1. For very large numbers and extremely small ones, these numbers can be placed in scientific notation in order to express them in a more concise or compact form.

2. Numbers in Scientific Notation can be used in a computation with far greater ease. This last advantage was more practical before the advent of calculators and their abundance.

3. In scientific fields, scientific notation is still used.

Numbers Greater Than 10

1. First locate the decimal and move it either right or left so that there are only one non-zero digit to its left.

2. The resulting placement of the decimal will produce the N part of the standard Scientific Notational Form.

3. Count the number of places that you had to move the decimal to satisfy step 1 above.

4. If it is to the left as it will be for numbers greater than 10, that number of positions will equal x in the general expression.

As an example, place the number 23419 in standard scientific notation?

1. Position the decimal so that there is only one non-zero digit to its left. In this case we end up with 2.3419

2. Count the number of positions we had to move the decimal to the left and that will be x.

3. Multiply the results of step 1 and 2 above for the standard form:

   So we have: 2.3419×10^4

How about numbers less than one?

Follow the same steps except in order to position the decimal with only one non-zero decimal to its left, we will have to move it to the RIGHT. The number of positions that we had to move it to the right will be equal to $-x$. In other words we will end up with a negative exponent. Negative exponents can be rewritten as values with positive exponents by taking the inversion of the number. For example:

10^{-5} can be rewritten as $^1/_{10^5}$.

Here is an example to consider: Express the following number in scientific notation:

0.436

1. Move the decimal to the right in order to satisfy the condition of having one non-zero digit to the left of the decimal. That will give us:

   4.36

2. Count the number of positions that we had to move it which was 1. That will equal $-x$ or $x = -1$

   And the expression will be 4.36×10^{-1}

What about numbers between 1 and 10?

In those numbers we do not need to move the decimal so the exponent will be zero. For example:

7.92 can be rewritten in notational form as: 7.92×10^{0}

Express the following numbers in their equivalent *Standard Notational Form*:

1. 123,876.3
2. 1,236,840.
3. 4.22
4. 0.000000000000211
5. 0.000238
6. 9.10

One of the advantages of this notation is the ability to compute with them in an easier fashion than with actual numerical equivalents.

Let's discuss how one would multiply with such notations. The general format for multiplying using scientific notation is as follows:

$(N \times 10^x)\ (M \times 10^y) = (N)\ (M) \times 10^{x+y}$

1. First multiply the N and M numbers together and express an answer.
2. Second multiply the exponential parts together by applying the *Law of Exponents*.
3. Finish by multiplying the two results together.

For example: $(3 \times 10^4)\ (1 \times 10^2)$

1. First $3 \times 1 = 3$
2. Second $(10^4)\ (10^2) = 10^{4+2} = 10^6$
3. Finally 3×10^6 for the answer

Another example: $(4 \times 10^3)\ (2 \times 10^{-4})$

1. First $4 \times 2 = 8$
2. Second $(10^3)\ (10^{-4}) = 10^{3+(-4)} = 10^{3-4} = 10^{-1}$
3. Finally 8×10^{-1} would be the answer

Express the product of the following:

1. $(3 \times 10^5)\ (3 \times 10^6) = ?$
2. $(2 \times 10^7)\ (3 \times 10^{-9}) = ?$

3. $(4 \times 10^{-6})\ (4 \times 10^{-4}) = ?$

As a footnote all answers in *Standard Notational Form*. That is where the *N* or *M* number has a value between 1 and 10. A computed number value that appears before the power of ten is greater than 10 or less than 1.

For example:

$$165 \times 10^{8}$$

has to have its decimal position adjusted. Move the decimal to the left two positions. When the decimal is moved to the LEFT, you would add a positive 1 to the exponent for **EACH** position that you moved the decimal to the left.

$$1.65 \times 10^{8+2} = 1.65 \times 10^{10}$$

When the decimal is moved to the RIGHT, you would add a negative 1 to the exponent for **EACH** position that you move the decimal to the right. For example:

$$0.0078 \times 10^{5}$$

requires the decimal to be moved 3 positions to the right. The answer would be:

$$7.8 \times 10^{5+(-3)} = 7.8 \times 10^{5-3} = 7.8 \times 10^{2}$$

Division of Exponentially Notated Numbers

Now let's discuss Division using scientific notation. The general format is as follows:

$$(N \times 10^{x}) / (M \times 10^{y}) = N/M \times 10^{x-y}$$

For example:

$$(6 \times 10^{5}) / (2 \times 10^{2}) =$$

1. Perform the division on the *N* and *M* numbers , $^{6}/_{2} = 3$
2. Perform the division on the exponential parts by subtracting the exponent in the lower number from the exponent in the upper number.

   $$10^{5} \div 10^{2} = 10^{5-2} = 10^{3}$$

3. Multiply the two results together, 3×10^{3}

Let's take another example, $(8 \times 10^{-3}) \div (2 \times 10^{-2})$

1. First $^8/_2 = 4$
2. The exponent is $-3 - (-2) = -1$. So, $10^{-3+2} = 10^{-1}$
3. Multiply the result of step 1 above with the result of step 2 to get the answer:

   4×10^{-1} or 4.0×10^{-1}

Express the quotients of the following:

1. $(3.45 \times 10^{8}) / (6.74 \times 10^{-2}) = ?$
2. $6.7 \times 10^{7} / 8.6 \times 10^{3} = ?$ (Be sure to express the answer in Standard Form)
3. $4.7 \times 10^{-2} / 5.7 \times 10^{-6} = ?$

Addition and Subtraction Using Exponential Notation

The important thing to remember about adding or subtracting *Scientific Notation* is **the exponents must be the same** before the math operation can be performed. The general format would be:

$$(N \times 10^{x}) + (M \times 10^{x}) = (N + M) \times 10^{x}$$

or

$$(N \times 10^{y}) - (M \times 10^{y}) = (N - M) \times 10^{y}$$

If the exponential equivalents do not have the same exponent then the decimal of one has to be repositioned so that its exponent is the same as all the rest of the numbers being added or subtracted. The reason for that is that when we add or subtract numbers we must line all the decimals up in the same position before we add or subtract columns of numbers. So for example:

$$(2.3 \times 10^{-2}) + (3.1 \times 10^{-3})$$

We recognize that the two exponents are not the same so either the exponent of the first number has to be changed to –3 or the exponent of the second number has to be changed to –2. It is arbitrary which one is changed. Let's change the first one.

$$2.3 \times 10^{-2}$$

Remember for each position to the RIGHT we add negative 1 to the exponent and for each position to the LEFT we add positive 1 to the exponent. In this case, we must reposition the decimal one position to the right so it becomes:

$$23. \times 10^{-2+(-1)} = 23. \times 10^{-3}$$

Now both numbers will have the SAME exponent value.

$$(23. \times 10^{-3}) + (3.1 \times 10^{-3}) = (23. + 3.1) \times 10^{-3} = 26.1 \times 10^{-3} = 2.61 \times 10^{-2}$$

Here is another example:

$$(4.2 \times 10^{4}) - (2.7 \times 10^{2}) =$$

Adjust the exponent of the second number this time. Remember it is arbitrary which one we adjust.

$$2.7 \times 10^{2}$$

must be repositioned two places to the LEFT. The exponent of the second number must be the same value as the exponent of the first number.

$$2.7 \times 10^{2} \text{ becomes } 0.027 \times 10^{2+2} = 0.027 \times 10^{4}$$

Now the problem reads

$$(4.2 \times 10^{4}) - (0.027 \times 10^{4}) = (4.2 - 0.027) \times 10^{4} = 4.173 \times 10^{4}$$

Now it is your turn,

Identify the sums or differences of the following:

1. $(8.41 \times 10^{3}) + (9.71 \times 10^{4}) = ?$
2. $(5.11 \times 10^{2}) - (4.2 \times 10^{2}) = ?$
3. $(8.2 \times 10^{2}) + (4.0 \times 10^{3}) = ?$
4. $(6.3 \times 10^{-2}) - (2.1 \times 10^{-1}) = ?$

Express the following numbers in their equivalent *Standard Notational Form*:

1. $123{,}876.3 = 1.238763 \times 10^{5}$
2. $1{,}236{,}840. = 1.23684 \times 10^{6}$
3. $4.22 = 4.22 \times 10^{0}$
4. $0.000000000000211 = 2.11 \times 10^{-13}$
5. $0.000238 = 2.38 \times 10^{-4}$

6. $9.10 = 9.10 \times 10^{0}$

Express the product of the following:

1. $(3 \times 10^{5})\ (3 \times 10^{6}) = 9 \times 10^{11}$
2. $(2 \times 10^{7})\ (3 \times 10^{-9}) = 6 \times 10^{-2}$
3. $(4 \times 10^{-6})\ (4 \times 10^{-4}) = 1.6 \times 10^{-9}$

Express the quotients of the following:

1. $3.45 \times 10^{8} / 6.74 \times 10^{-2} = 5.12 \times 10^{9}$
2. $6.7 \times 10^{7} / 8.6 \times 10^{3} = 7.80 \times 10^{3}$ (Be sure to express the answer in STANDARD form)
3. $4.7 \times 10^{-2} / 5.7 \times 10^{-6} = 8.25 \times 10^{3}$

Identify the sums or differences of the following:

1. $(8.41 \times 10^{3}) + (9.71 \times 10^{4}) = 10.55 \times 10^{4} = 1.055 \times 10^{5}$
2. $(5.11 \times 10^{2}) - (4.2 \times 10^{2}) = 0.91 \times 10^{2} = 9.1 \times 10^{1}$
3. $(8.2 \times 10^{2}) + (4.0 \times 10^{3}) = 48.2 \times 10^{2} = 4.82 \times 10^{3}$
4. $(6.3 \times 10^{-2}) - (2.1 \times 10^{-1}) = -14.7 \times 10^{-2} = -1.47 \times 10^{-1}$