[24]D. W. E. Green, "1991 TB_1," IAU Circular 5368 (1991).
[25]In retrospect, it probably would have been sufficient to track only at the sidereal (stellar) rate in this case, because of the brightness of 1991 TB_1. For targets which are significantly fainter than the field stars which would be used as positional references, however, this technique would be valuable as it would increase the signal-to-noise ratio in the image of the asteroid. In fact, for faint asteroids, it might be preferable for the actual simultaneous exposures to be trailed in this fashion for the same reason, especially if the parallax measurement is to be made by the overlay method rather than by using computer software to derive the pixel coordinates of the asteroidal and stellar images.
[26]D. W. E. Green, "1992 AC," IAU Circular 5426 (1992), "1992 AC," IAU Circular 5442 (1992), "1992 AC," IAU Circular 5474 (1992).
[27]D. King-Hele, *Observing Earth Satellites* (Van Nostrand Reinhold, New York, 1983).
[28]D. Tattersfield, *Orbits for Amateurs with a Microcomputer, Volume II* (Wiley, New York, 1987), pp. 118–154.
[29]W. Livingston and D. Talent, "Stalking geosats with a camera," Sky & Telescope **80**, 319 (1990).

Negative mass can be positively amusing

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Some insights into gravitation and mechanics, as well as some surprises, result from considering the dynamics of negative mass particles.

Negative masses may be unphysical, but several examples are given of how they can be instructive. In these examples, the dynamics and gravitational interactions involving negative mass particles produce surprising (and amusing) predictions about the motions of the particles. That these apparently impossible motions are, in fact, compatible with physical laws leads to some insights about dynamics and gravitation.

At the outset it must be made clear that the "mass" of a body has several different roles in mechanics: (i) The "inertial mass" of a body refers to its resistance to acceleration; it is the m in $\mathbf{F}=m\mathbf{a}$. (ii) The "gravitational," or "passive gravitational mass," governs how strongly gravity pulls on the body; it is the m in $\mathbf{F}=-m\nabla\Phi$, where Φ is the gravitational potential. (iii) There is also "active gravitational mass," which determines the strength of the gravitational field generated by a body.[1] In standard physical theory all three "masses" are identical.

The equivalence of inertial and passive gravitational mass, in particular, has been verified to high accuracy, and is known by the name "the principle of equivalence." It is this principle that allows us to cancel the masses in Newton's second law, and in the force law for gravity, so that the gravitational acceleration is given by $\mathbf{a}=-\nabla\Phi$. Since no reference to the mass appears, this equation predicts that (in a vacuum) feathers and rocks fall in the same way. We will assume that the equivalence principle holds, so that our negative mass particles have both negative inertial mass, and negative passive gravitational mass.

The first situation to be considered is the fall of a negative mass particle when it is released from rest. The particle, like rocks and feathers, must of course fall downward.[2] For the negative mass particle, of course, the gravitational force on the particle is upward. The particle accelerates (downward) in the direction opposite to the force acting on it due to the particle's negative inertial mass. More interesting than the free-fall of the negative mass particle is how it must be constrained, e.g., before it is released, to prevent it from falling. Since the gravitational force on it is upward, the support force to prevent the particle from falling must be downward. For example, the particle could be tethered by a string and the other end of the string could be pulled downward. This would give us a child-with-a-balloon configuration, with the negative mass particle suspended above the child, who feels an upward pull on the string. There is, of course, a crucial difference between the negative mass particle and an actual balloon. If the string breaks, a balloon—to the child's chagrin—would accelerate upward, whereas the negative mass particle would fall downward.

This becomes even stranger if we replace the child as the agent of downward force by a positive mass particle. Suppose that we have a negative mass particle of -1 kg at the top of the string in the earth's uniform field. Let us put a $+1$-kg mass at the bottom of the string. The $+1$-kg mass pulls downward on the string with a force of 9.81 N, the -1-kg pulls upward on the string with the same force. The string remains under a tension of 9.81 N and the whole configuration—positive mass, negative mass, and string—remains fixed in position falling neither downward or upward. We have created an "antigravity glider." This is acceptable, if not sensible, since the total mass of the configuration is zero.[3] But the consequence of cutting the string may be less acceptable. If the string is cut both particles fall!

We can further offend our intuition with the antigravity glider. Suppose we release from rest the ± 1-kg mass configuration with 10-N tension in the string. At the bottom of the string the 10-N upward tension force will not quite cancel the downward 9.81-N gravitational force. There will be a net upward force of 0.19 N which will result in an upward acceleration of the $+1$-kg mass by 0.19 m s^{-2}. At the top of the string, the 10-N downward tension and the upward 9.81-N gravitational force leave a net 0.19-N downward force which, acting on a -1-kg mass, results in a 19 m s^{-2} *upward* acceleration. The entire configuration will therefore initially accelerate upward. Since the bottom and the top particles will maintain their separation, the tension in the string will remain 10 N, and the configuration will continue to accelerate upward at the same rate. The rate of acceleration, of course, can be adjusted, or its direction reversed, by adjustments to the tension in the string. Despite initial appearances, the "free acceleration" would not violate physical law. Since the total (inertial) mass of the system is zero, the system can accelerate without the action of any external force. And there is no problem with energy conservation. As the system accelerates upward the increase of gravitational potential energy of the bottom (positive mass) particle is offset by the decrease in gravitational potential of the top (negative mass) particle. Since the particles always have the same velocity the positive kinetic energy of the bottom mass and the negative kinetic energy of the top mass will always add to zero.

The potential of this configuration seem limitless. But harnessing its advantages would require solving technological problems other than simply finding negative masses. One such problem emerges when we consider moving a negative mass particle from place to place. If we have the particle tethered to an elastic string and we tug on the string, the particle responds to the tug by perversely accelerating in the opposite direction, the direction away from the tug. This motion of the particle will stretch the string increasing the tugging force, and increasing the acceleration of the particle in the "wrong" direction. That there should be this awkward instability is inherent in the mathematics of simple harmonic motion. If we characterize the elasticity of the string by spring constant k, the natural frequency for the motions of a particle of mass $-m$, where m is positive, would be $\sqrt{k/(-m)} = \pm i\sqrt{k/m}$. Velocities and accelerations of the particle would in general grow with time t as $\exp \sqrt{k/m}\, t$. In a realistic situation (although the meaning of "realistic" is not quite clear in this context) the string would quickly reach its failure strength and break, and the particle would fly off at high speed.

Clearly, a simple tug on a string is an unsuccessful strategy for bringing a negative mass particle nearer. Pushing the particle away with, say, a pole *would* get the particle going in the right direction, but quick reflexes (that is, monitoring and feedback loops) would be needed to get the pole out of the way so that the particle's acceleration did not grow exponentially in time. Great vigilance would be needed, in fact, at all times with negative mass particles. Ordinary containers which ultimately rely on elastic forces could not easily be used. If the negative mass particle ever impinged on the elastic walls it would, in short order, burst through. Any equilibrium due to elastic forces would be unstable. Convenient confinement (without the action of monitoring and feedback loops) would require a position of stable equilibrium in a force field. One such position for a negative mass particle would be at a local minimum of the gravitational potential since, by the equivalence principle, the particle's motions due to gravity are the same as for a positive-mass particle.

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[1]Here, we are primarily concerned with inertial and passive gravitational mass, but further amusement can be had from a particle which also has negative "active" gravitational mass. Such a particle will be gravitationally attracted to, but will gravitationally repel, a positive mass particle, and will chase it forever. See Alan P. Lightman, William H. Press, Richard H. Price, and Saul A. Teukolsky, *Problem Book in Relativity and Gravitation* (Princeton U. P., Princeton, 1975), Prob. 13.20.

[2]The equivalence principle has rather the status of a coincidence in Newtonian gravitation theory, but is crucial and central to metric theories of gravitation like general relativity. The fact that a negative mass particle would fall downward is thus required in general relativity. In this connection it is worth mentioning that all the gravitational phenomena described in this article can be described equally well in general relativity. The apparent paradoxes are in no way caused by the shortcomings of Newtonian gravitational theory.

[3]There is nothing fundamentally physically wrong with a weightless string. Such strings can be described in general relativity and are, in quite different contexts, of some interest as "cosmic strings." It is well known that cosmic strings do not induce curvature in the surrounding space-time. [See, for example, T. M. Helliwell and D. A. Konkowski, "Cosmic strings: Gravitation without local curvature," Am. J. Phys. **55**, 401–407 (1987).] The string therefore has no gravitational pull, so it should not be surprising (and it is straightforward to verify) that it has no weight.

THE JOY OF INSIGHT

Indeed, the joy of insight is something very important. I myself must say, if I look back at my life as a scientist and a teacher, I think the most important and beautiful moments were when I say, "ah-hah, now I see it a little better," and it is not necessarily when I myself have done something. When I hear a seminar or when I hear a good speaker, then I say, "ah, now I see," this is this joy of insight which pays for all the trouble one has had in this career.

Victor F. Weisskopf, in *Quarks, Quasars, and Quandaries*, edited by Gordon J. Aubrecht (AAPT, College Park, Maryland, 1987), p. 13.